

Notes on graphing $y = A \sin (Bx + C) + D$ and $y = A \cos (Bx + C) + D$

1. Sketch a reference graph of one period of the mother function on $[0, 2\pi]$. You have to memorize these, and showing range and domain, x and y intercepts.
2. Factor B out of the argument (the angle) by writing it as $Bx + C$ as $B(x + C/B)$.
3. Write down the values of the amplitude (A), the period ($2\pi/B$), and the horizontal shift ($-C/B$).
4. Determine the endpoints of the function for one cycle (one period) by solving for x when the argument $Bx + C = 0$ and $Bx + C = 2\pi$. (Think about why this makes sense: Endpoint $x = 0$ moves to $-C/B$ and $x = 2\pi$ moves to $(2\pi - C)/B$.)
5. Take a moment to consider if the wave frequency is increased (shorter period, occurs when $B > 1$) or decreased (longer period, occurs when $B < 1$).
6. Begin to sketch the new function, USING PENCIL.

Draw one complete period between the two new endpoints. Sine and cosine have symmetry, so find the halfway point between the new endpoints. That is where the middle intercept will go.

7. Consider the amplitude. If $|A| > 1$, it is stretched vertically, which you indicate by showing the maximum extending beyond 1 up to A and the minimum from -1 down to $-A$.

If $|A| < 1$, the graph is obviously compressed vertically.

And if A itself is negative, the graph flips over the x-axis.

Sketch this amplitude effect over the graph you just drew.

8. Finally, if there is a raising or lowering of the entire graph, it will be seen in D. The easiest way to make this clear is to relabel the y-axis so the midline that was the x-axis ($y = 0$) is now the line $y = D$. The maximum will be $A + D$. The y-intercept will also move accordingly. It is the value of the function at $x = 0$.

Example: In the screenshot an example is shown of the transformation of $\sin(x)$ to $y = -\sin(2x + \pi/3) + 1$.

I used Desmos at <https://www.desmos.com/calculator> (thanks to Kyle). The issue with this and every online calculator is that you don't get radians expressed as multiples of π . Still, it serves our needs wonderfully to see the steps of the transformation.

The period is $2\pi/B = 2\pi/2$.

The shift of $\pi/4$ is found by factoring out 2 in the argument: $(2x + \pi/2) = 2(x + \pi/4)$.

The endpoints are found by solving $2x + \pi/3 = 0$ and $2x + \pi/3 = 2\pi$.

One cycle thus goes from $-\pi/6$ (about $-1/2$) to $5\pi/6$ (about 2.6).

The entire graph is then raised 1 unit up, so the midline is the $y = 1$ axis.

