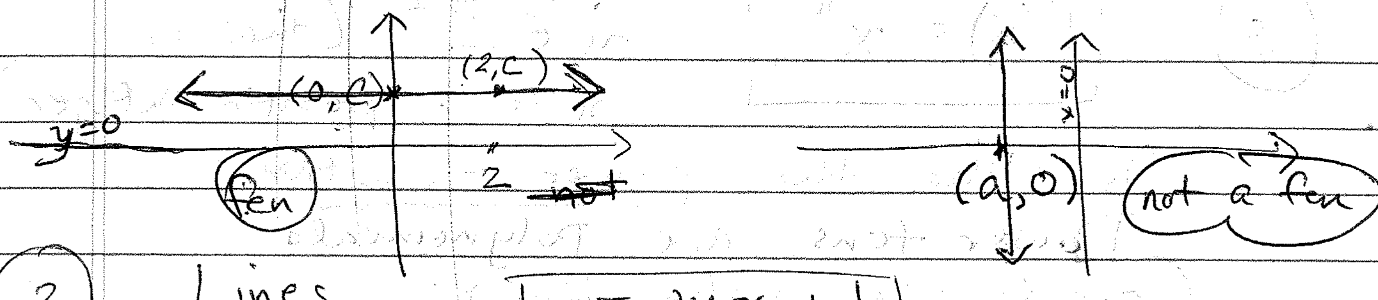
Thumbnails

Graphing

Math 108 - Chapter 4 - Family of graphs

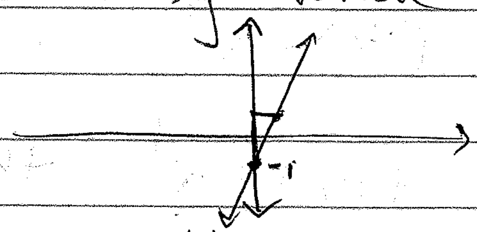
Family 1. Lines $y = c$ for all x $\quad$ $x = a$ for all y

where a, c are constants.



2. Lines $\boxed{y = mx + b}$

where m is the slope of the line, positive, negative, or zero, and b is the y-intercept of the line, that is, the value of y when $x = 0$.



$y = \frac{2}{1}x - 1$
m is positive
so slope goes up

For lines, rewriting as $y = f(x)$ shows the functional nature of the graph.

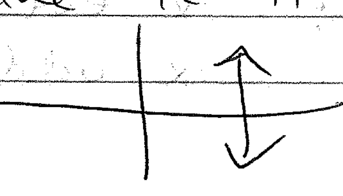
$f(x) = c$ and $f(x) = mx + b$

pass the vertical line test, but

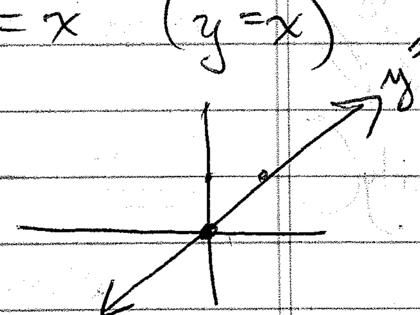
$x = a$

is not a function b/c, clearly, it fails the VLT.

vertical line b/c it is a vert. line



Finally, we refer to $f(x) = x$ $(y = x)$
as the "identity fcn".



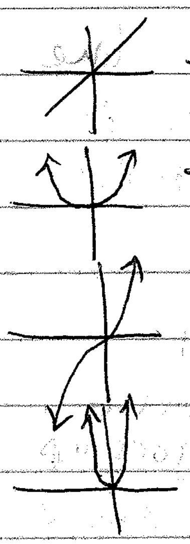
$y = \frac{1x}{1}$ $\quad m = 1 \quad b = 0$

$\boxed{y = 1x + 0}$

(3) $\boxed{f(x) = x^n,}$ $\quad n \in \mathbb{Z}^+$ (that is, n is a positive integer)

is called the "power function"
Power fcns. are polynomials.

Consider these values for n:



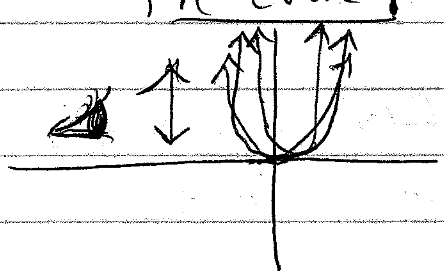
$n = 1 \quad f(x) = x^1$ the identity fcn (line)

$n = 2 \quad f(x) = 1x^2$ the parabola

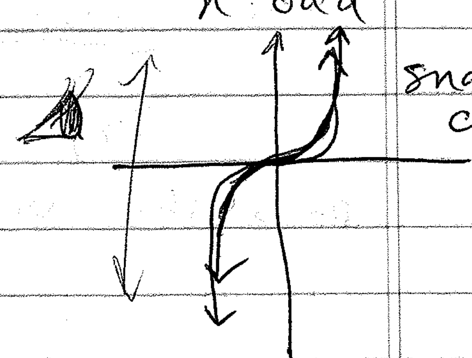
$n = 3 \quad f(x) = x^3$ the cubic

$n = 4 \quad f(x) = x^4$ the quartic

$\boxed{n \text{ even}}$ $\qquad\qquad$ n odd



U-shaped curves



snake curves

Domain: $x \in \mathbb{R}$ true of all polynomials
n odd or even

~~Range~~

Range: x even, $y \geq 0$
x odd $-\infty < y < \infty$, $\mathbb{R}$

$f(x) = x^n$ features

Domain? $x \in \mathbb{R}$ since power fcns. are polynomials

Range? Depends on n odd or even
n Even: $y \geq 0$ n Odd: $\mathbb{R}$

Fcn Even or odd?: n even $\Rightarrow f(x)$ even
n odd $\rightarrow f(x)$ odd

symmetric about y-axis $\Leftarrow$ Ex. $f(x) = x^2$
$f(-x) = (-x)^2 = x^2$ $\Rightarrow \boxed{f(x) = f(-x)}$

symmetric about origin $\Leftarrow$ Ex $f(x) = x^3$
$f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3$ $\Rightarrow \boxed{f(-x) = -f(x)}$

Sum up
$f(x) = x^{odd}$ $f(-x) = -f(x)$ an odd fcn
$f(x) = x^{even}$ $f(x) = f(-x)$ an even fcn.

One-to-one? $f(a) = f(b) \Rightarrow a = b$ $\boxed{\text{If } f(a)=f(b) \text{ then } a \text{ is equal to } b}$

Even power fcns. are clearly not one-one since they fail the horizontal line test.

Ex $f(1) = 1^{even} = 1 = f(-1)$
$f(-1) = 1^{odd} = -1 \neq f(1)$

Look carefully at graphs on pp 93-97
+ at the "Important Ideas" which
sum up the features of power fcns.

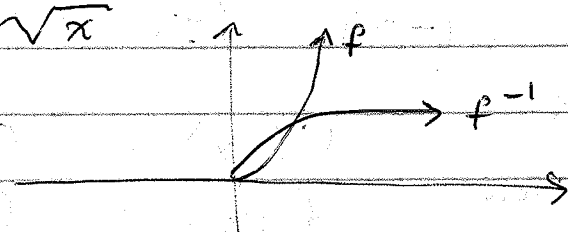
Root functions $\quad f(x) = x^{\frac{1}{n}}$

First + foremost, these are the inverse
fcns. of the power fcns. on the
appropriate domain.

9/30 Root fcns + Rational (reciprocal) fcns.

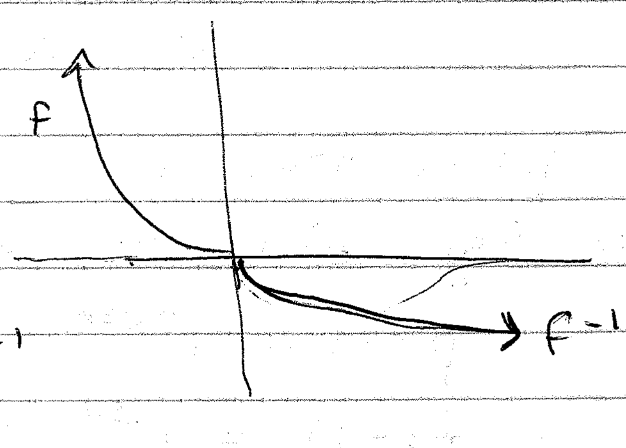
The inverse fcn. of $f(x) = x^2$ that exists on $[0, \infty)$
is $f^{-1}(x) = \sqrt{x}$

Dom f = Range f^{-1}



The inverse fcn. of $f(x) = x^2$ on $(-\infty, 0]$
is $f^{-1}(x) = -\sqrt{x}$.

Dom f = Range f^{-1}



Range f = Dom f^{-1}

$y = x^2 \longrightarrow$ on $\mathbb{R}$ / negatives

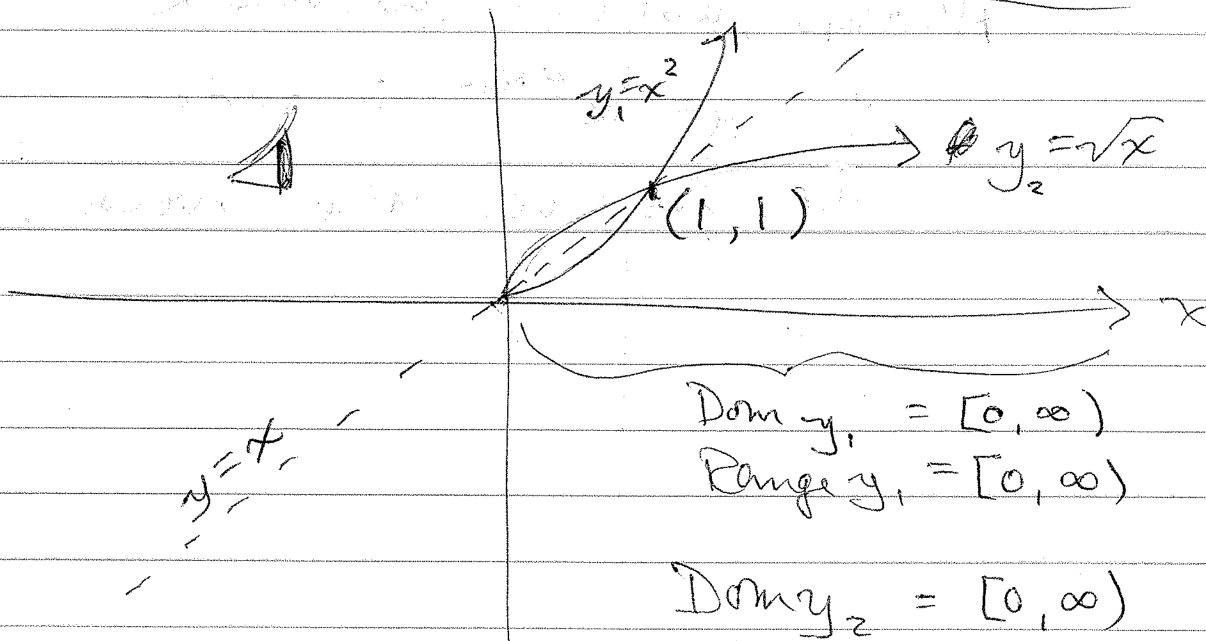
$x = y^2$

$\sqrt{x} = \sqrt{y^2}$

$\pm\sqrt{x} = y$

For f + f^{-1}, we find dom f $=$ range f^{-1} and range f $=$ dom f^{-1}

Power fcns have root fcns as inverses



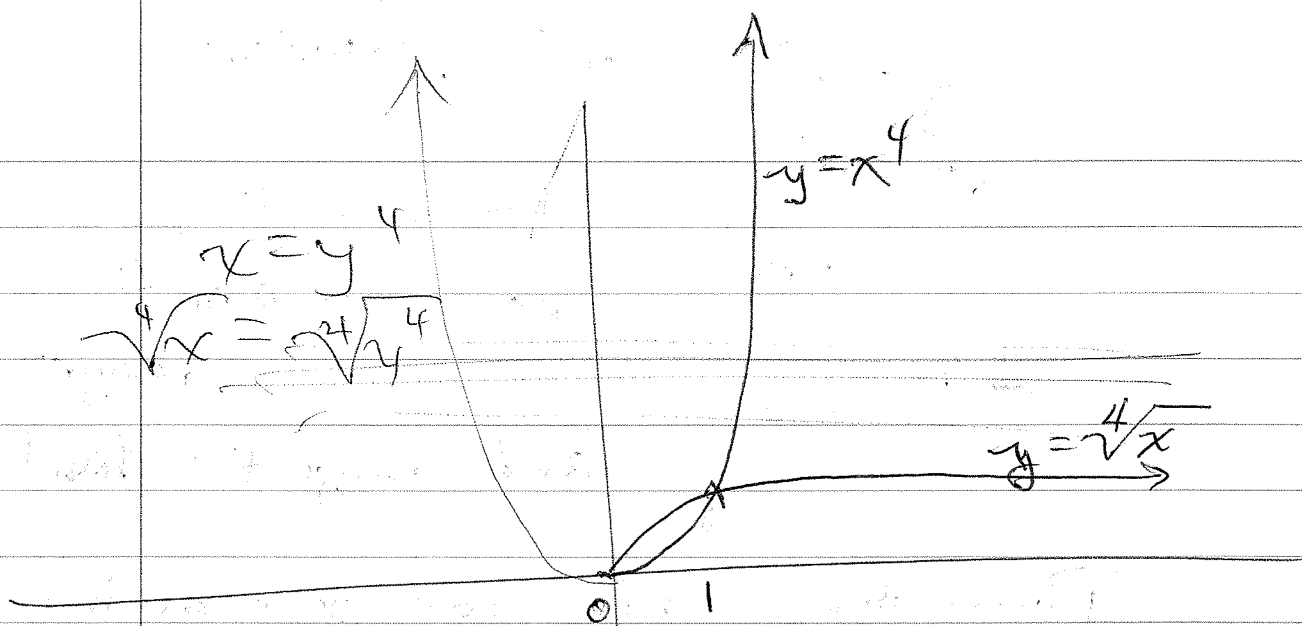
Dom y_1 $= [0, \infty)$
Range y_1 $= [0, \infty)$

Dom y_2 $= [0, \infty)$
Range y_2 $= [0, \infty)$

"Root fcn" means square root fcn, but "root-fcns" is a catch-all for $\boxed{y = \sqrt[n]{x}}$

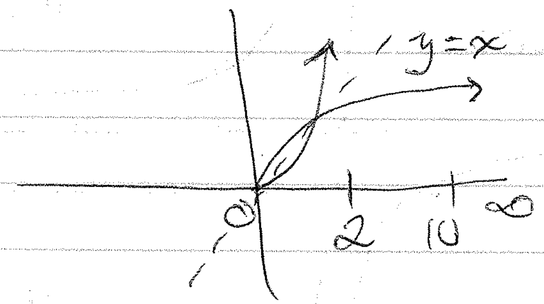
$$y_1 = y_2$$
$$x^2 = \sqrt{x}$$

soln to the system $\boxed{x = 0, 1}$ "pts of intersection"



Always restrict the domain of $y = x^{even}$ in order to discover the inverse.

Draw the axis of symmetry $y = x$ to see the inverse nature of $f(x) = x^2$ + $g(x) = \sqrt{x}$



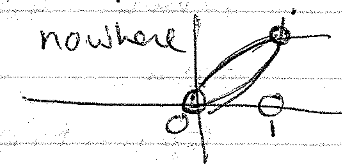
[2, 10]

Where do $y = x^2$ & $y = \sqrt{x}$ intersect on $[0, \infty)$? On $(0, 1)$?

(0,0) + (1,1)

open interval

nowhere



<u>Reciprocal fcn</u> - the simplest kind of kind rational fcn function

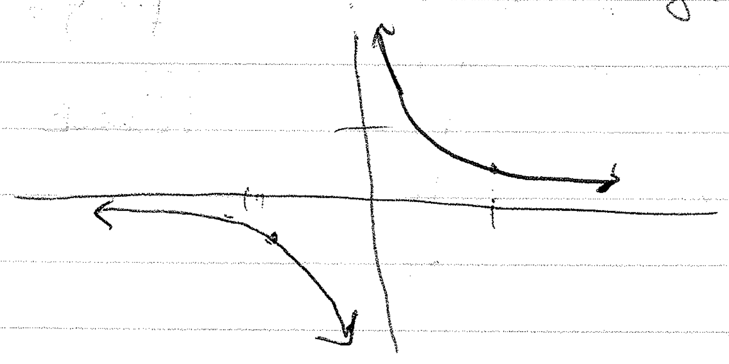
i.e., fcns of the form $\frac{f(x)}{g(x)} = y$ functions

are "rational ~~fcn~~ fcns". If $f(x) = 1$ and $g(x) = x$, we get the simplest ~~of these,~~

$\boxed{\frac{1}{x} \text{ reciprocal of } x}$ $\boxed{y = \frac{1}{x}} = \frac{f(x)}{g(x)}$

Dom: $x \neq 0$ Range: ?

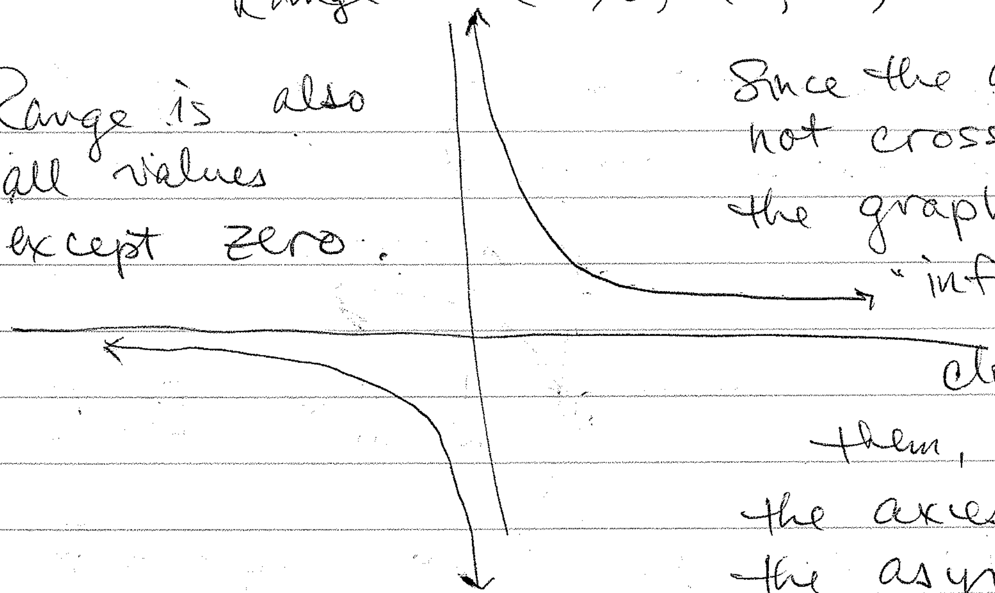
x	y
-2	-1/2
-1	-1
0	∞
1	1
2	1/2



x	y
-1	-1
-1/2	-2
-1/4	-4
0	∞
1/4	4
1/2	2
1	1

$$\text{Dom} = (-\infty, 0) \cup (0, \infty) \quad \text{or} \quad x \neq 0$$
$$\text{Range} = (-\infty, 0) \cup (0, \infty) \quad \text{or} \quad y \neq 0$$

Range is also all values except zero.



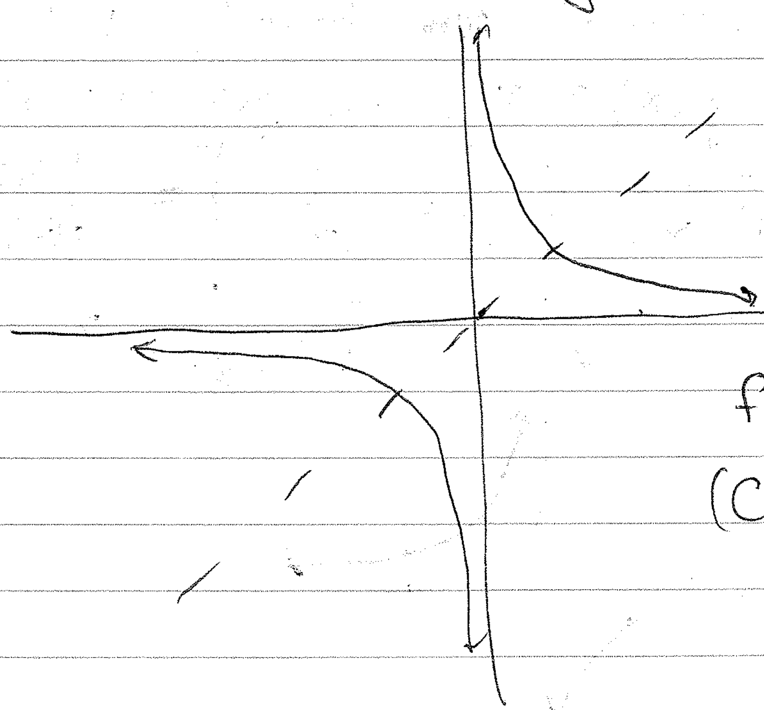
Since the axes are not crossed but the graph gets "infinitesimally close" to them, we say the axes are the asymptotes.

When you name the asymptotes of a fcn, you must name them as an equation:

$x = 0$ vertical asymptote

$y = 0$ horizontal asymptote

Draw the line $y = x$ on this graph.



Notice that the fcn serves as its own inverse.

$f(x) = \frac{1}{x}, \quad f^{-1}(x) = \frac{1}{x}$

(Check $f \circ f^{-1}$, $f^{-1} \circ f$)

Finally, $f(x) = \frac{1}{x}$ is odd because

$$f(-x) = \frac{1}{-x} = -\frac{1}{x} = -f(x)$$

It's one-one because if we assume

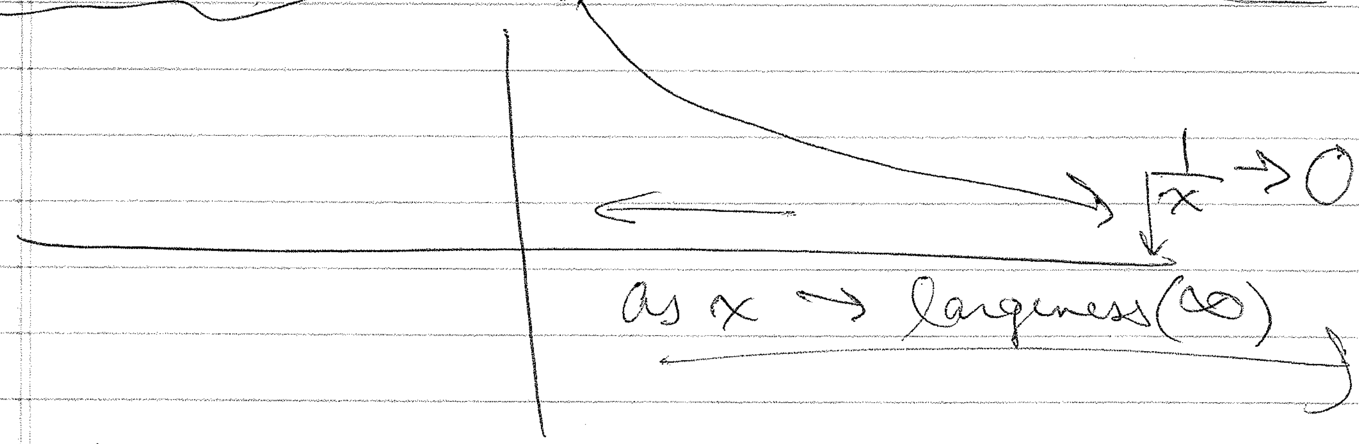
If $f(a) = f(b)$
(that is, $\frac{1}{a} = \frac{1}{b}$)

then clearly $a = b$.

We can anticipate the idea of the limit of a fcn. by studying $y = \frac{1}{x}$

as x gets very large or very small

$$y = \frac{1}{\text{large}} \approx 0 \quad , \quad y = \frac{1}{\text{tiny}} \approx \infty$$

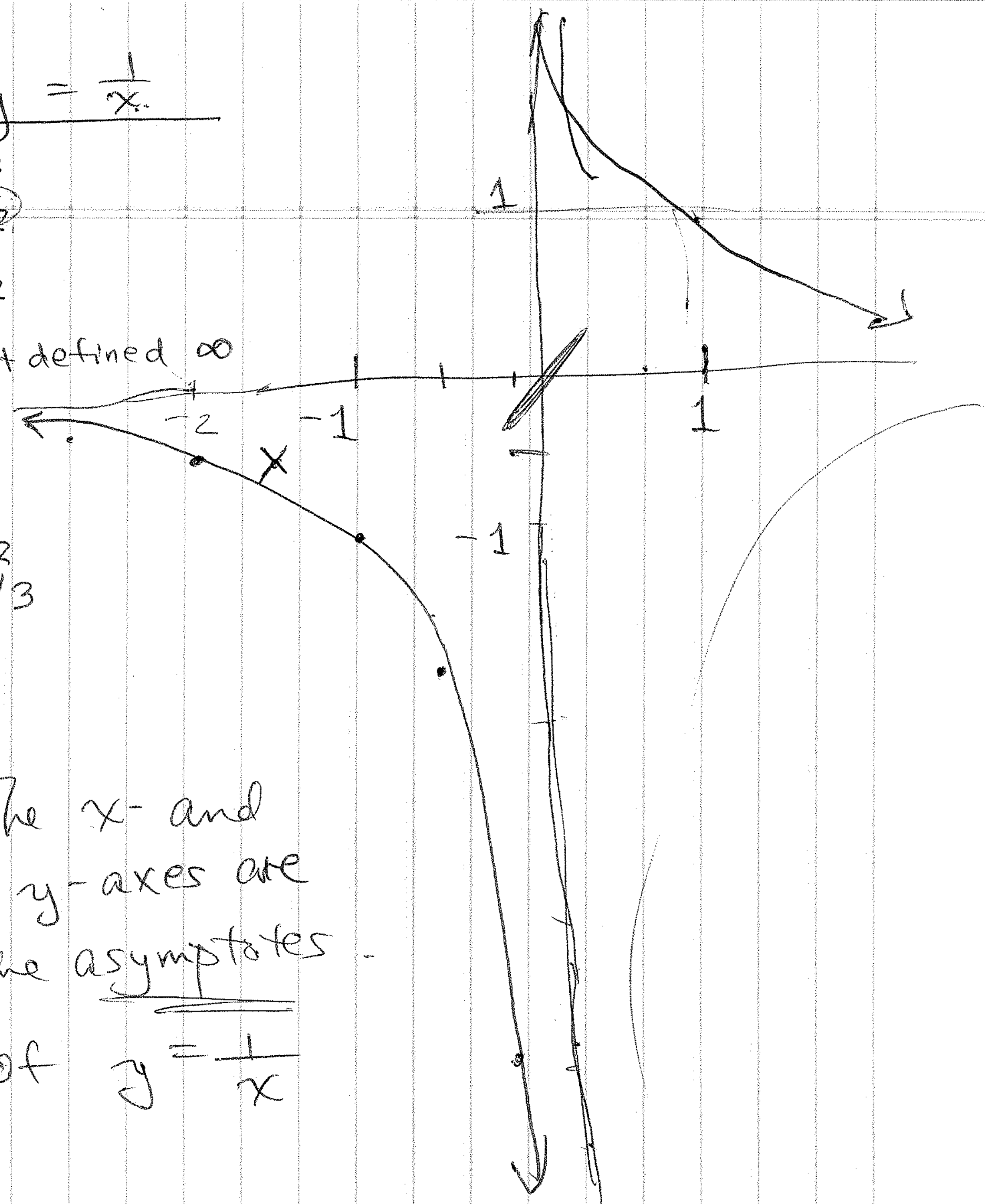


Domain: $y = \frac{1}{x}$ $(-\infty, 0) \cup (0, \infty)$

Range:

Transformations

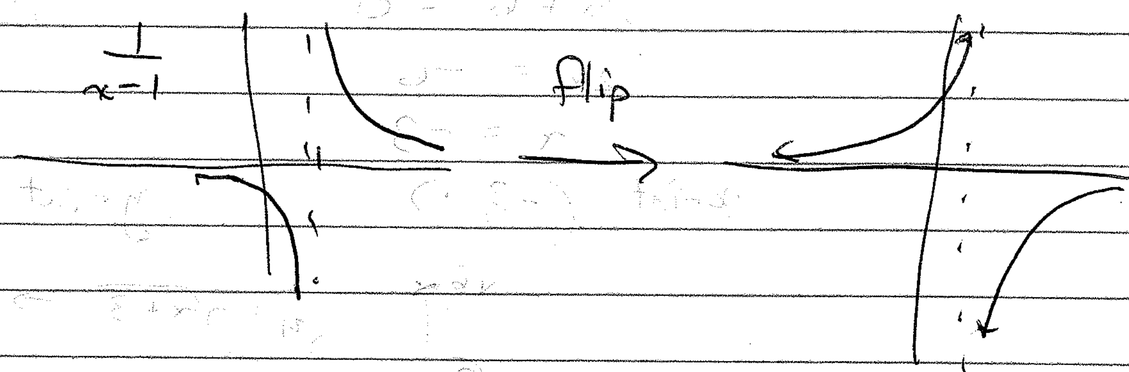
x	$y = \frac{1}{x}$
-3	$-1/3$
-2	$-1/2$
-1	-1
$-1/2$	-2
$-1/4$	-4
0	not defined ∞
$1/4$	4
$1/2$	2
1	1
2	$1/2$
3	$1/3$



The x- and y-axes are the asymptotes of $y = \frac{1}{x}$

$$y = \frac{1}{-x+1} = \frac{1}{-(x-1)}$$

$\frac{1}{x-1}$



10/2

So far, we've only graphed rigid transformations of our three basic fcns. (power, root, reciprocal)

Non-rigid transformations entail a multiplier of x.

$$y = 2x^2 \qquad y = \sqrt{3x}$$

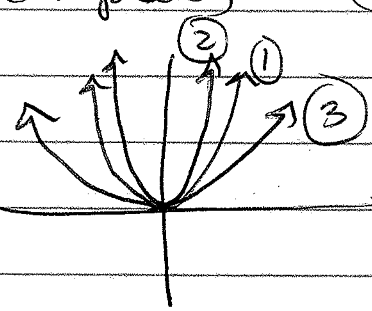
Change their shape

$$y = \frac{-4}{x}$$

in terms of their wideness or narrowness.
(compress) (stretch)

② $y = 3x^2$

③ $y = \frac{x^2}{4} = \frac{1}{4}x^2$



p.159 Do Comp. Check

$y = \sqrt{2x+6} = 0$

$2x + 6 = 0$

$2x = -6$

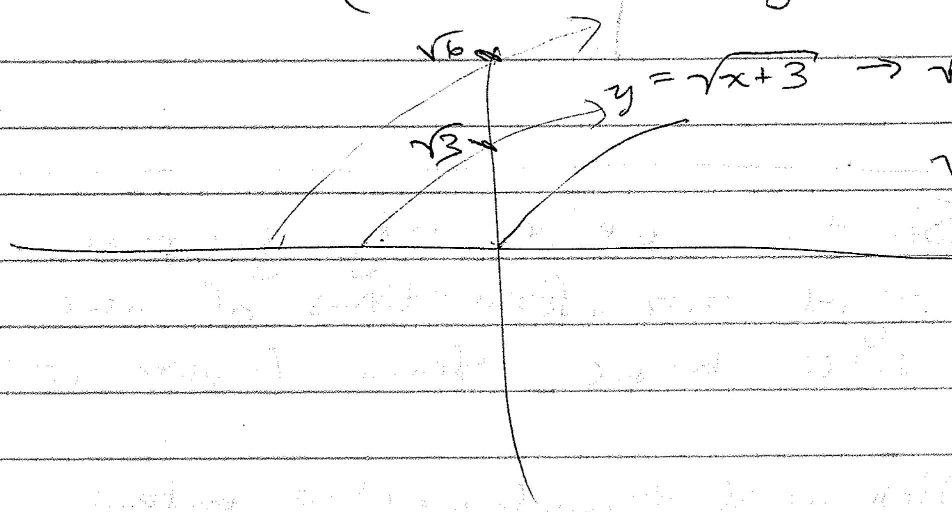
$x = -3$

x-int $(-3, 0)$

$y = \sqrt{2\cdot 0 + 6} =$

$= \sqrt{6}$

y-int $(0, \sqrt{6})$



$y = \sqrt{x+3} \rightarrow \sqrt{0+3} = \sqrt{3}$

$\sqrt{2}\sqrt{3} = \sqrt{6}$

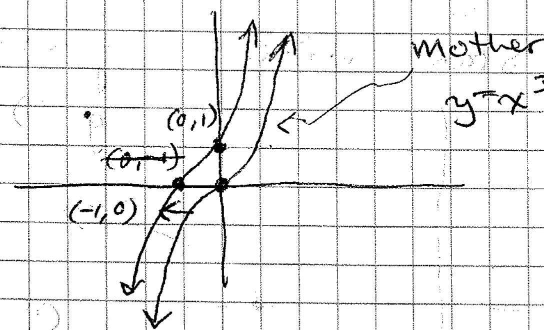
4.2

#1a $g(x) = (x+1)^3$

y-int $g(0) = (0+1)^3 = 1$

root (x-int) $0 = (x+1)^3$

$x = -1$

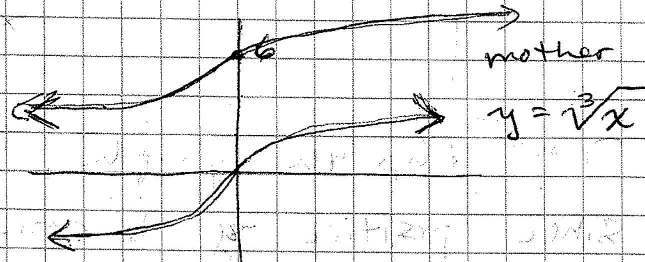


#1b $g(x) = \sqrt[3]{x} + 6$

$g(0) = \sqrt[3]{0} + 6 = 6$

$0 = \sqrt[3]{x} + 6$

$x = (-6)^3 = -216$ (too far out to graph)

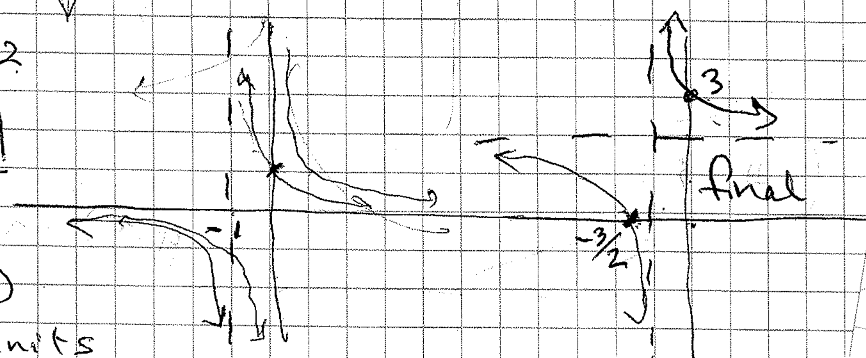


#1c $g(x) = \frac{1}{x+1} + 2$

Vert. Asymptote $\boxed{x = -1}$

Horiz. Asymptote $y=0$ (x-axis)

Now move it up 2 units (check y-int, any roots, change in asymptotes?)



$g(0) = 1+2 = 3$

$0 = \frac{1}{x+1} + 2$

$-2 = \frac{1}{x+1}$

$x+1 = -\frac{1}{2}$

$x = -\frac{3}{2}$

Notice the change in the horizontal asymptote:

$g(x) = \frac{1}{x+1} \to 0$ as $x \to \infty$

But $\frac{1}{x+1} + 2 \to 0+2$ as $x \to \infty$

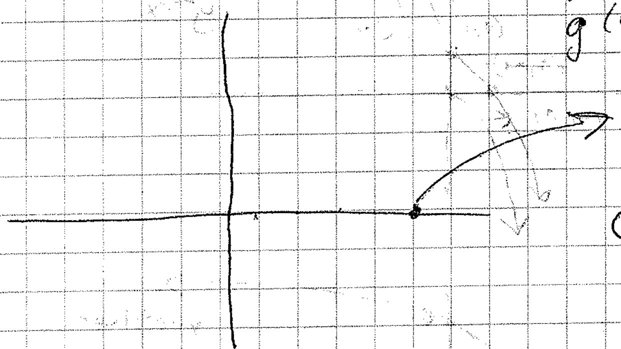
So HA is $\boxed{y=2}$ eqns of lines

while VA is still $\boxed{x=-1}$

$g(x) = -\sqrt{x-5}$

Start with $g(x) = \sqrt{x-5}$

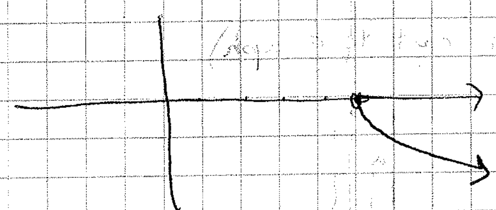
$g(0)$ = undefined since Dom g $x \geq 5$ (so there's no y-int)



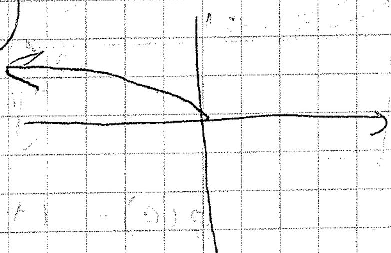
$0 = \sqrt{x-5}$

$x = 5$ is the x-intercept (root)

Now flip the graph, that is, reflect it over x-axis since positive y becomes negative



#1i



$g(x) = \sqrt{-x}$ Dom: $-x \geq 0$

$x \leq 0$

#1l $g(x) = \sqrt{-x+1} - 2$

"Mother fcn" is $\sqrt{-x} = y$ (above)

What happens to the root?

$0 = \sqrt{-x+1} - 2$

$2 = \sqrt{-x+1}$

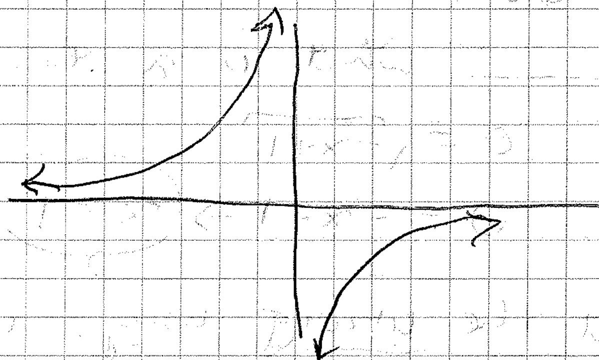
$4 = 1 - x \longrightarrow x = -3$ is new root (x-int)

What is y-int?

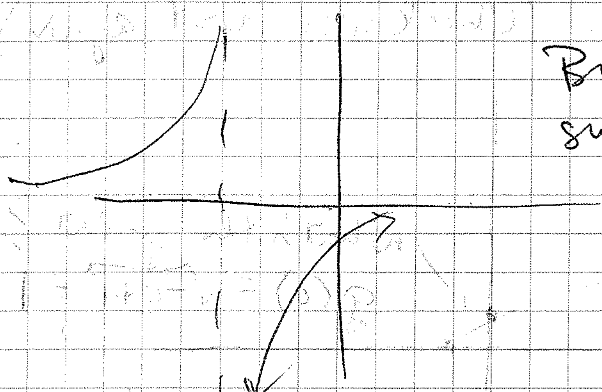
$g(0) = \sqrt{0+1} - 2 = -1 = y$ is new y-int

(#1 n) $g(x) = \dfrac{1}{-x+3}$ 1

Mother is $y = \dfrac{1}{-x}$



What is the shift? 3 right or left?
We know from last problem that this is deceptive. If we went left it would look like this



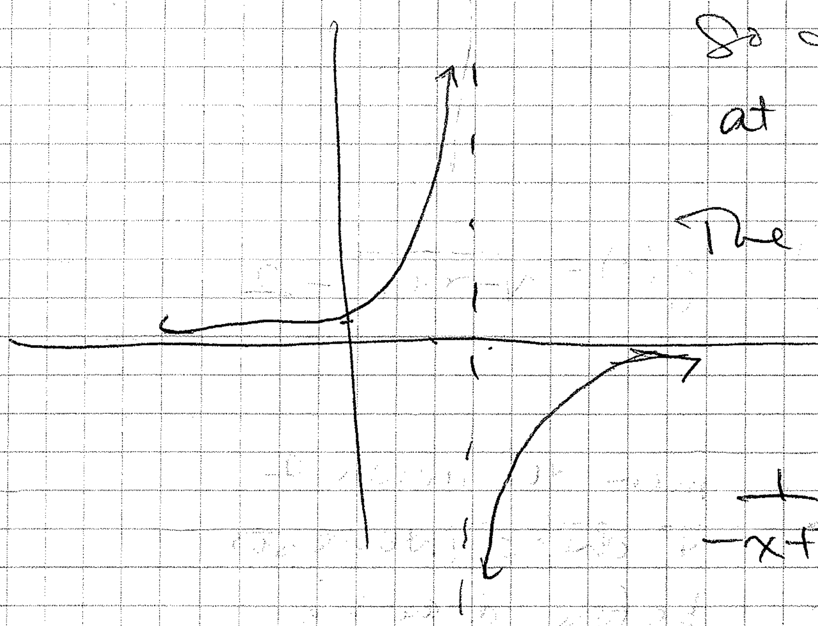
But what is the vert. asymp. supposed to be?

$$g(x) = \frac{1}{-x+3}$$

VA: $-x+3 \neq 0$

$x \neq 3$

So the graph has the VA at $x = 3$



The horiz. asymp. is still $y=0$ (x-axis)

since

$$\frac{1}{-x+3} \longrightarrow 0 \text{ as } x \to \infty \text{ or } -\infty$$