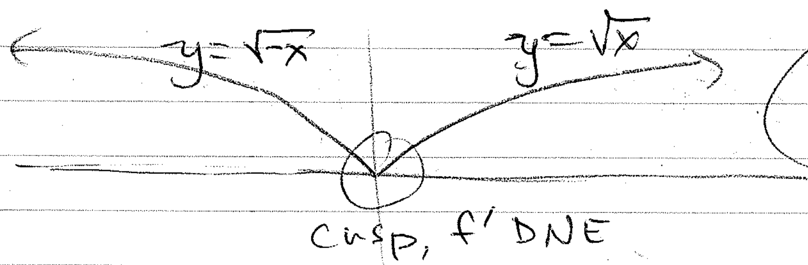
⑩ 3/24 We continued with #2h today but first we considered a different function:

$$f(x) = \sqrt{|x|}$$ (~~next page~~)

Consider first $y = \sqrt{|x|} = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$

Dom: $x \in \mathbb{R}$



Reflect graph over y-axis when x is changed to −x

What is $\frac{dy}{dx}$ of $y = \sqrt{|x|}$?

Differentiate both parts:

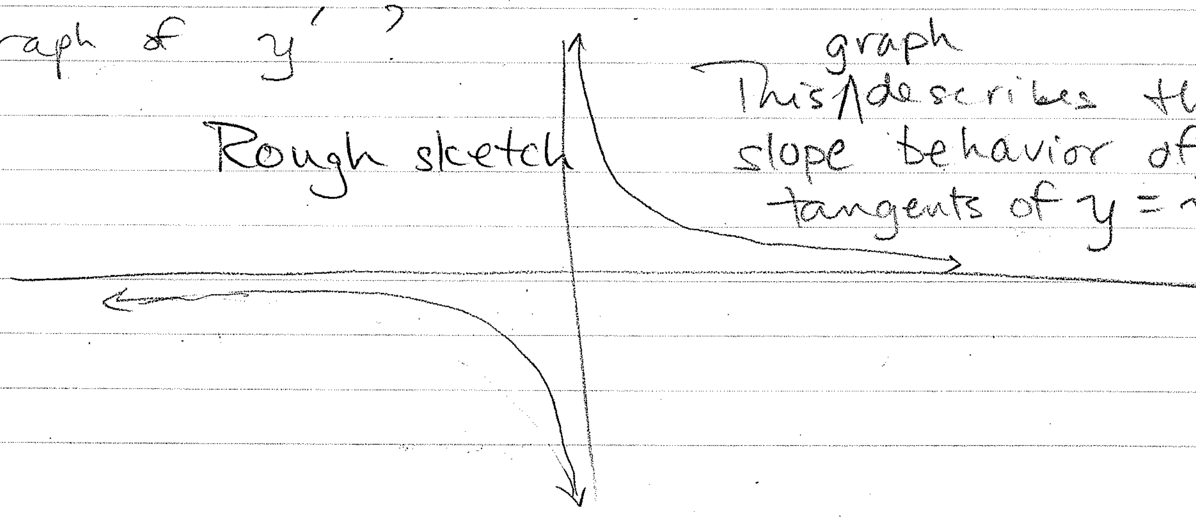
$$y = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$$

$$y' = \begin{cases} \frac{1}{2}x^{-1/2}, & x > 0 \\ \frac{1}{2}(-x)^{-1/2}(-1), & x < 0 \end{cases} = \begin{cases} \frac{1}{2\sqrt{x}}, & x > 0 \\ \frac{-1}{2\sqrt{-x}}, & x < 0 \end{cases}$$

chain rule

Crit values? where y' DNE, i.e., $x = 0$

Graph of y'?



Rough sketch

This graph describes the slope behavior of tangents of $y = \sqrt{|x|}$

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#2h

$f(x) = \sqrt{4x^2+3} = (4x^2+3)^{1/2}$ Dom: $x \in \mathbb{R}$

$f'(x) = \frac{1}{2}(4x^2+3)^{-1/2}(8x) = \dfrac{4x}{\sqrt{4x^2+3}}$

Crit values?: $\dfrac{4x}{\sqrt{4x^2+3}} = 0$ at $x = 0$

That's it

No restrictions on this.

y-int $f(0) = \sqrt{3}$

To see if $x = 0$ is an extreme or an inflection pt., either check $f'(-1)$ & $f'(1)$:

$f'(-1) = \dfrac{-4}{+} < 0$, $f'(1) = \dfrac{4}{+} > 0$

Hence, $f \downarrow$ on $(-\infty, 0)$; $f \uparrow$ on $(0, \infty)$

$(0, \sqrt{3})$ is a local min.

What about concavity? Second derivative helps out here:

Quotient rule: $f''(x) = \dfrac{4\sqrt{4x^2+3} - (4x)(\frac{1}{2})(4x^2+3)^{-1/2}\cdot(8x)}{(\sqrt{4x^2+3})^2}$

$= \dfrac{4(4x^2+3)^{1/2} - 16x^2(4x^2+3)^{-1/2}}{4x^2+3}$ DIY LCD

$= \dfrac{4(4x^2+3) - 16x^2}{(4x^2+3)^{3/2}} = 0$

$= \dfrac{16\cancel{x^2} + 12 - 16\cancel{x^2}}{(4x^2+3)^{3/2}} = 0$ nowhere, no POI

In fact $f'' > 0$ on $\mathbb{R}$

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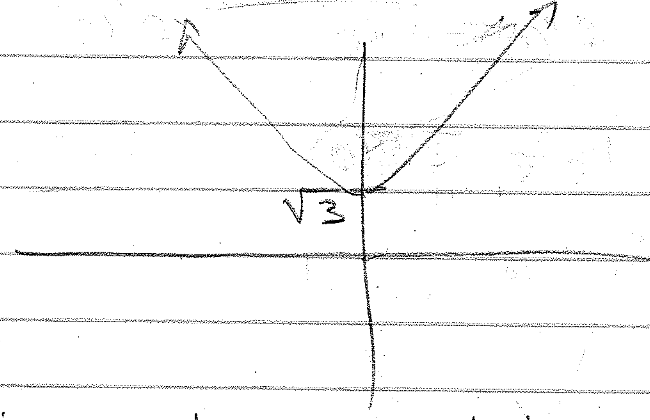
#2h continued

Since $f'' > 0$ on all x, it's concave up.



This is not a parabola or any other polynomial.

What categories of graphs do we have?

1. Polynomial fcns. $y = 2x^3 - 3x^2 + 4x - 5$
2. Root fcns $y = 4x^{2/3} - x^{1/2}$
3. Log fcns
4. Exponential fcns

$y = x + \ln x$

$y = 5xe^x$

2. a) $f(x) = \frac{1}{3}x^3 - 4x + 6$ Dom: $x \in \mathbb{R}$

$f(0) = 6$ $f(x) = 0$ at ??

Crit pts, $f \uparrow, \downarrow$ inflection

$f'(x) = x^2 - 4 = 0$ at $x = \pm 2$

$f''(x) = 2x$; $f''(-2) = -4 < 0$ c. down

$f''(2) = 4 > 0$ c. up

So local max at $x = -2$

& local min at $x = 2$

$f''(x) = 0$ at $x = 0$ POI

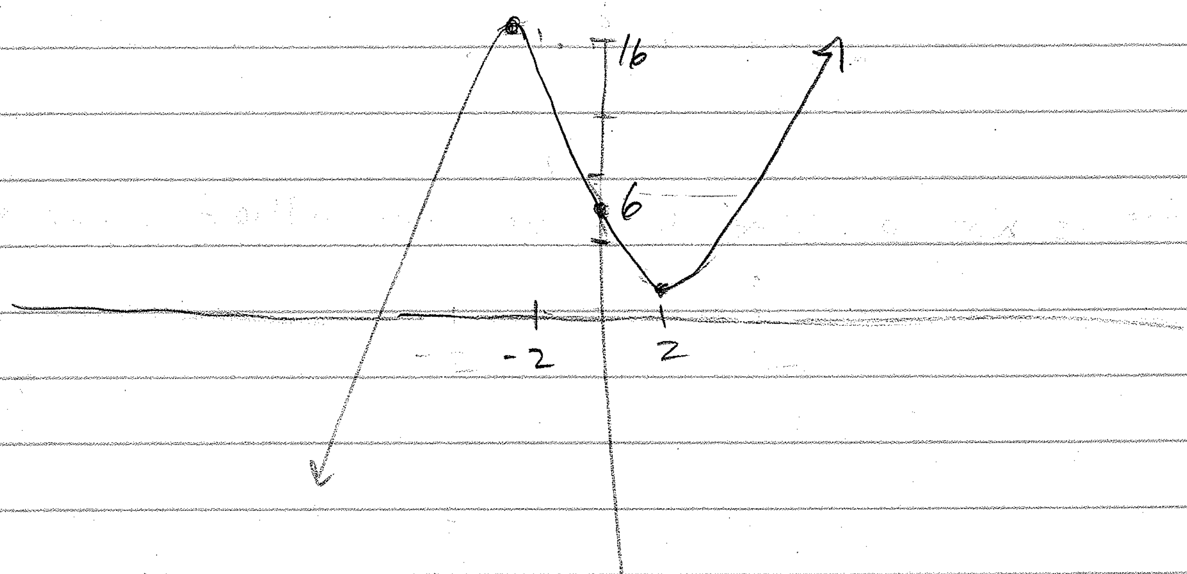
(13) Test values: $x = -3, 0, 3$ into f' to find $\uparrow, \downarrow$

$f\uparrow$ max $f\downarrow$ min $f\uparrow$

$c_1 = -2$ $\qquad$ PoI $= 0$ $\qquad$ $c_2 = 2$

$f'(-3) = 5 > 0$ $\qquad$ $f'(0) = 4 > 0$ $\qquad$ $f'(3) = 5 > 0$

Calculated $f(-2) = 16\frac{2}{3}$, $f(0) = 6$, $f(2) = \frac{2}{3}$



Watch your y-axis scale. It might need to be compressed, like it is here, since our local max is $f(-2) = 16\,^2/_3$

Notice that there is but one real root, somewhere left of $x = -2$. The other 2 roots (deg $= 3$) are complex.