Practice Problems for 220 Final

1. Evaluate:

$$\lim_{x\to 1}\frac{3x^2 - x - 2}{x^2 + 5x - 6}$$

$$\lim_{x\to\infty}\frac{3x^4 + x - 2}{-2x^4 + 5x}$$

$$\lim_{x\to 3^+}\frac{x^2 - 5x}{x - 3}$$

2. Differentiate:

$$f(x) = 2x^5 + \sqrt[3]{x} + \frac{4}{x}$$

$$f(x) = e^{x^2+4x}$$

$$f(x) = \frac{2x^3 + x}{x^5 - 1}$$

$$f(x) = (3x^4 + 7x - 9)\ln x$$

$$f(x) = \sqrt{4x^2 + \ln(x^2 + 1)}$$

$f(x,y) = 3y^8 + x^2y + e^{xy} - 5$, find $f_x$, $f_{xx}$, and $f_{xy}$.

3. Integrate:

Find $f$ such that $f(1) = 4$ and $f'(x) = 2x + \frac{1}{\sqrt{x}} + 3$

$$\int (2x^3 + 3x)(x^4 + 3x^2)^8 dx$$

$$\int_0^1 \frac{7x^3}{x^4 + 1} dx$$

$$\int xe^{-3x} dx$$

$$\int 3x^3(x^2 + 1)^5\, dx$$

4. Find the area between the $x$-axis and the graph of $f(x) = 3x^2 - 12$ on $[1, 4]$. (Find the ACTUAL area, not the SIGNED area.)

5.

a) An investment of \$15000 is made. It grows with continuous compounding at a 7% rate per year for 14 years. What is its value at the end of that time?

b) An investment is made as a continuous money flow of \$5000 per year of 7 years. Interest of 8% per year is compounded continuously. What is the present value of the investment?

c) An investment of \$1000 is made. Interest is compounded quarterly with an annual interest rate of 5%. How long will it take for the money to triple?

6. The profit from the sale of $x$ dryers and $y$ washers is given by $P(x, y) = 200x - 0.2x^2 + 300y - 0.8y^2 - 500$. 100 total machines can be made. How many of each should be made to maximize profit?

7. A hairstylist is doing 80 haircuts per month at \$26 each. A study has shown that for each decrease in price of \$2.50, the number of haircuts per month will increase by 10.
a) What price should be charged to maximize monthly revenue?
b) At that price, how many haircuts will be sold in a month?

8. Suppose the demand for a product is given by $q = 1500 - 0.05p^2 - 0.2p$ where $p$ is the price per unit and $q$ is the quantity sold.
a) Find the elasticity function $E(p)$.
b) If price is \$100/item, would a slight increase in price lead to an increase or decrease in revenue?
c) Find the price that maximizes revenue using your elasticity function.

9. Charles buys a computer. He estimates the worth of his computer by $W(t) = 18(t - 10)^2 + 200$, where $W$ is the worth (\$) of computer $t$ years since the purchase date for $0 \leq t \leq 10$.
a) How much did Charles pay for the computer?
b) "The worth of the computer decreases for the entire 10 years." Justify this statement using Calculus.
c) At what rate is the worth of the computer changing two years after it was purchased?
d) Find the average worth of the computer over its 10-year lifespan.