Review Problems for MATH 220 – Exam 3 (Sections 17-24)

<u>Absolute Extrema / Optimization</u>

The population in a small town from the beginning of the year 2000 to the beginning of the year 2010 is modeled by the function $P(t) = t^3 - 15t^2 + 63t + 10{,}000$, where $P(t)$ represents the population in the town t years after the start of 2000 (in other words, $t = 0$ is the start of the year 2000). Answer the following questions.

a) On what time interval(s) was the population in the town declining? Justify.

b) Find the maximum population in the town over the 10-year interval.

Given $f(x) = \frac{4x}{x^2+9}$, find the absolute maximum and minimum on $[0, 5]$.

Given $f(x) = \frac{1-x}{x^2+3x}$, find the absolute extrema on $[1, 4]$.

Given $f(x) = 2x^3 - 6x^2 - 48x + 7$, find the absolute maximum and minimum on $[-6, 8]$.

The cost to build q items is $6000 + 5q + 0.01q^2$. In order to sell q items, the price will need to be $p(q) = 20 - \frac{q}{4}$. Find the quantity that will maximize profit.

A rectangular enclosure is to be built next to a river. There will be no fence on the river side of the enclosure. The cost for the material for the side parallel to the river is $6.00 per foot. The material for the sides perpendicular to the river costs $2.00 per foot. There is a budget of $240.00 for the fence. What dimensions of the fence result in the largest enclosed area?

A movie theater has a seating capacity of 525 people. With the ticket price at $10, average attendance at a movie has been 375 people. Management has decided to lower admission prices to boost attendance. A market survey indicates that for each two dollars the price of a ticket is lowered, average attendance will increase by 100. What ticket price will maximize revenue from ticket sales?

A 12 foot piece of wire is cut into two pieces. The first piece is bent into a square and the second piece is bent into an equilateral triangle. Where should the wire be cut to minimize the combined area of the two resulting shapes?

A Norman window is in the form of a rectangle surmounted by a semicircle. Find the dimensions of the window that will admit the most light.

Curve sketching

Suppose $f(x) = 3x^3 - 9x + 7$. Graph.

Suppose $g(x) = \frac{x^2}{3x-2}$. If so, $g'(x) = \frac{3x^2-4x}{(3x-2)^2}$ and $g''(x) = \frac{8}{(3x-2)^3}$. Graph.

Suppose $f(x) = \frac{x^2-1}{x^3}$. If so, $f'(x) = \frac{(3-x^2)}{x^4}$ and $f''(x) = \frac{2(x^2-6)}{x^5}$. Graph.

Suppose $h(x) = x^{\frac{2}{3}}\left(\frac{5}{2} - x\right)$, so $h'(x) = \frac{5(1-x)}{3x^{\frac{1}{3}}}$ and $h''(x) = \frac{-5(1+2x)}{9x^{\frac{4}{3}}}$. Graph.

Suppose $f(x) = \frac{2x^2-6x}{3x^2-8x-3}$. Find the equations of all horizontal/vertical asymptotes.

Suppose $f(x) = 2x(x-4)^3$. Find all local extrema. Classify each as a local maximum or local minimum.

Elasticity of demand

The demand function for a particular product is $q = \sqrt{50 - p^2}$. At a price of $3.00, is demand elastic or inelastic? If price is increased slightly, does revenue go up or down?

Suppose $3p + \sqrt{q} = 800$ indicates the relationship between price *p* and demand *q* for some commodity.

a) Find $E(p)$.

b) Does the commodity in this example exhibit elastic or inelastic demand at $p = \$70$. Justify your answer using the result from part (a).

c) Hence, to increase revenue, should the company raise or lower the price?

d) Use $E(p)$ to determine the price that should be charged to maximize revenue. Give your answer to the nearest dollar.

Suppose the demand for a product is given by $q = 1500 - 0.05p^2 - 0.2p$.

a) Find $E(p)$.

b) If the price is $100/item, would a slight increase in price lead to an increase or decrease in revenue?

c) Find the price that maximizes revenue using $E(p)$.

MULTIPLE CHOICE

The product of a pair of real numbers that have a sum of 40 is $P(x) = x(40 - x)$ where x is one of the numbers. This product has a maximum when

a) $x = 60$

b) $x = 40$

c) $x = 0$

d) $x = 20$

The sum of a pair of positive real numbers that have a product of 9 is $S(x) = x + \frac{9}{x}$ where x is one of the numbers. This sum has a minimum when

a) $x = 0$

b) $x = 3$

c) $x = 6$

d) $x = 9$

Which of the following is not true?

a) any even-degree polynomial has at least one absolute extreme on the reals

b) any odd-degree polynomial has an absolute extreme on the reals

c) the absolute extreme of a function can occur at an endpoint

d) a local minimum can also be an absolute minimum

If a function is continuous on a closed interval $[a, b]$, then…

a) any local extreme will also be an absolute extreme

b) it will have both an absolute maximum and an absolute minimum on $[a, b]$

c) it cannot have the same absolute maximum value and absolute minimum value

d) the absolute extremes must occur at $x = a$ and $x = b$

The absolute minimum of $f(x) = \ln(x+2)$ on $[-1, \infty)$ is:

a) $x = 0$

b) $x = -1$

c) Nowhere

d) $x = -2$

The absolute maximum of $f(x) = x^3 - 6x^2 + 9x - 8$ on $[0, 5]$ occurs at:

a) $x = 5$

b) $x = 1$

c) $x = 0$

d) $x = 3$

Suppose we want to maximize the volume $V = xyz$ subject to the conditions $x = y$ and $xy + xz + yz = 100$. The volume expressed in terms of x is:

a) $V = \frac{x(100-x^2)}{2}$

b) none of these

c) $V = x(50 - 2x^2)$

d) $V = 6x^3$

Which of the following statements is true?

a) The absolute minimum of a function can occur at an endpoint.

b) a local minimum can be an absolute maximum

c) the absolute minimum of a function cannot be a local minimum

d) a local maximum always occurs at an endpoint

A function with an absolute minimum on an interval I

a) must have a local minimum on I

b) does not need to be continuous on I

c) cannot have an absolute maximum on I

d) must be continuous on I

A function f has an absolute maximum on an interval provided that

a) f does not have a minimum on the interval

b) the interval is open and f is continuous on the interval

c) the interval is closed and f is continuous on the interval

d) f is differentiable on the interval

The function $f(x) = e$ has the property that

a) every point is an absolute maximum and an absolute minimum

b) it has no absolute maximum and no absolute minimum

c) it has no critical points

d) $x = \ln 1$ is its only critical number

If $f'(c) = 0$, then

a) f has absolute extrema at c

b) f has a local maximum or a local minimum at c.

c) f has a relative maximum or a relative minimum at c.

d) c is a *candidate* for local extrema