Math 108: Exam 3 **Name** Key KM

Spring 2016 **Instructor** _____

Page	1	2	3	4	Total	Course Points
Points	24	24	32	20	100	150
Score						

- Calculators are not permitted for this test.
- Show all work unless the problem requires only a short answer.
- There are problems on both front and back of the pages.
- If you need scrap paper, ask your instructor. You may not use your own. If you do use scrap paper, make sure to hand it in at the end of the exam.

1. (24 points) Solve each of the following inequalities for x. Express all answers using **interval notation.**

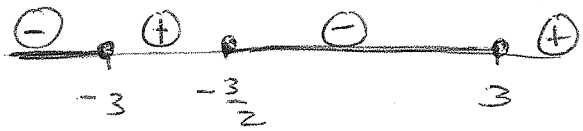
1. (a) $2x^3 + 3x^2 - 18x - 27 \leq 0$

$x^2(2x+3) - 9(2x+3) \leq 0$

$(x^2-9)(2x+3) \leq 0$

$(x+3)(x-3)(2x+3) \leq 0$

Zeroes: $x = -3, 3, -3/2$



test $x = -4, -2, 0, 4$

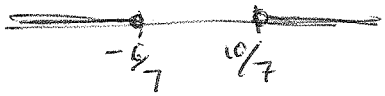
$\boxed{(-\infty, -3] \cup [-\tfrac{3}{2}, 3]}$

(b) $-6|7x-2| \leq -48$

$|7x-2| \geq 8$

$7x-2 \geq 8$ or ≤ -8

$x \geq 10/7$ or $x \leq -6/7$



$\boxed{(-\infty, -6/7] \cup [10/7, \infty)}$

(c) $3 - 2(x-5) > 8(x-5) + 12x$

$3 - 2x + 10 > 8x - 40 + 12x$

$13 + 40 > 10x + 12x$

$53 > 22x$

$x < \frac{53}{22}$

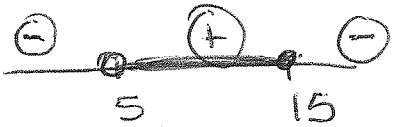
$\boxed{(-\infty, 53/22)}$

(d) $\frac{3x-5}{x-5} \geq 4$

$\frac{3x-5}{x-5} - 4 \geq 0$

$\frac{3x-5-4(x-5)}{x-5} \geq 0$

$\frac{-x+15}{x-5} \geq 0$



$\boxed{(5, 15]}$

2. (16 points) Graph the solution set to the following system of inequalities. *Algebraically find the vertices of the system and label them on your graph.*

$$\begin{cases} y > 3|x-2| - 1 \\ -x + y \le 1 \end{cases}$$

$3(x-2) - 1 = x + 1$

$3x - 6 - 1 = x + 1$

$2x = 8,\ y = 5$

$x = 4$

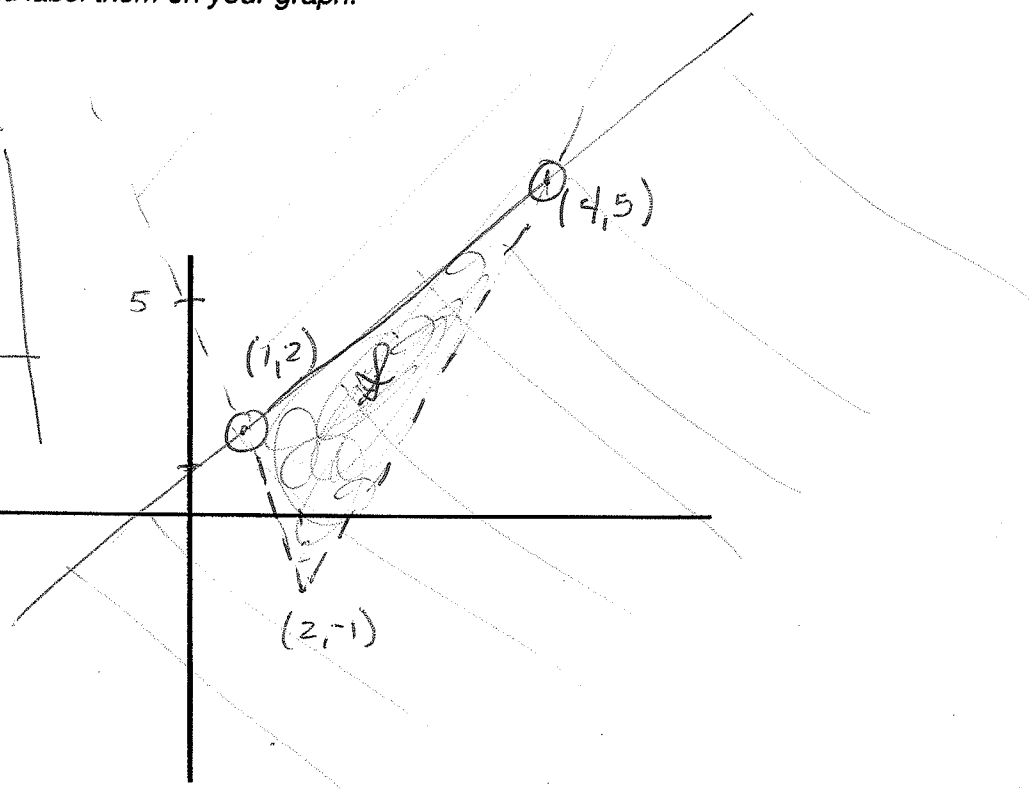
$3(-x+2) - 1 = x + 1$

$-3x + 6 - 1 = x + 1$

$-4x = -4$

$x = 1$

$y = 2$

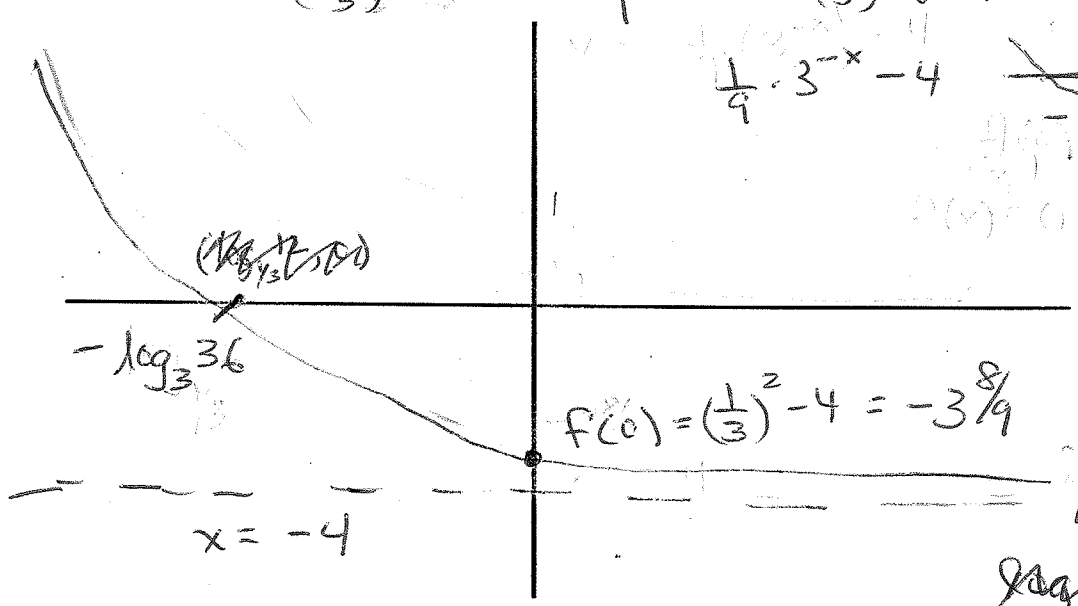


3. (8 points) Graph $f(x) = \left(\frac{1}{3}\right)^{x+2} - 4$ on the axes provided below. Label all intercepts and asymptotes.

$\left(\frac{1}{3}\right)^{x} = 3^{-x}$

$\left(\frac{1}{3}\right)^{2}\left(\frac{1}{3}\right)^{x} = \frac{1}{9} \cdot 3^{-x}$

$\frac{1}{9} \cdot 3^{-x} - 4$



root $\left(\frac{1}{3}\right)^{x+2} - 4 = 0$

$\frac{1}{9}\left(\frac{1}{3}\right)^{x} = 4$

$\frac{1}{3}^{x} = 36 \rightarrow 3^{-x} = 36$

$-x \log_3 3 = \log_3 36$

$\boxed{x = -\log_3 36}$

4. (16 points) Solve each system of equations. Express your answers as ordered pairs.

a) $\begin{cases} x = 3y + 6 & (1) \\ 3y = 14 - 9x & (2) \end{cases}$

$3y = 14 - 9(3y + 6)$

$3y + 27y = 14 - 54$

$30y = -40$

$y = -4/3$

$x = 2$ (sub y into (1) or (2))

$\boxed{(2, -4/3)}$

b) $\begin{cases} 2x - 5y = 7 \\ -6x + 15y = -21 \end{cases}$

$(3)(2x - 5y) = 7(3) \rightarrow 6x - 15y = 21$

$-6x + 15y = -21$

$0 = 0$

Soln: all (x, y) satisfying either eqn. $\left(x, \frac{2x-7}{5}\right)$

c) $\begin{cases} 4x + y = 2 \\ x^3 - 2 + y = 0 \end{cases}$

$2x = 5y + 7$

$\boxed{\frac{2x-7}{5} = y}$

$y = -4x + 2$

$x^3 - 2 + (2 - 4x) = 0$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0, 2, -2$

$y = 2, -6, 10$

$\boxed{(0,2) \quad (2,-6) \quad (-2,10)}$

5. (16 points) Evaluate each of the following expressions:

$\log_8 1 =$ 0

$\log_4\left(\frac{1}{16}\right) =$ -2

$5^{\log_5 11} =$ 11

$8^{\log_2 7} =$ 7^3

$2^{3\log_2 7} \leftarrow$

$\downarrow$

$2^{\log_2 7^3}$

$\log_4 128 =$ $7/2$

$4^x = 128$

$2^{2x} = 128$

$2x = 7$

$\log_{12}(144) = \log_{12} 36 + \log_{12} 4 =$ 2

$\log_3 81 = \frac{\log_7 81}{\log_7 3} =$ 4

$6 \ln e^{1/2} = 6 \ln \sqrt{e} =$ $3 \cdot 1 = 3$

$\parallel$

$\frac{1}{2} \cdot 6 \ln e$

6. (20 points) Solve each of the following equations for x.

a) $4^{2x+3} = 32^{x-1}$

$2^{2(2x+3)} = 2^{5(x-1)}$

$4x + 6 = 5x - 5$

$11 = x$

$\boxed{x = 11}$

Chk: $4^{22+3} \overset{?}{=} 32^{11-1}$

$2^{2(25)} \overset{?}{=} 2^{5(10)}$

$2^{50} = 2^{50}$ ✓

b) $\ln(4x + 13) = 3$

$e^3 = 4x + 13$

$\frac{e^3 - 13}{4} = x$

$\boxed{x = \frac{e^3 - 13}{4}}$

c) $\log_3 x + \log_3(x - 8) = 2$

$\log_3(x^2 - 8x) = 2$

$x^2 - 8x = 3^2$

$x^2 - 8x - 9 = 0$

$(x - 9)(x + 1) = 0$

$\boxed{x = 9}$, $\cancel{-1}$

Chk: $\log_3(9) + \log_3(1) = 2$

$2 + 0 = 2$

d) $6^{x-2} = 7^x$

$\log_6 6^{x-2} = \log_6 7^x$

$x - 2 = x \log_6 7$

$x - x\log_6 7 = 2$

$x(1 - \log_6 7) = 2$

$\boxed{x = \frac{2}{1 - \log_6 7}}$