Math 108-01 Quiz 5 (Do at home) ____

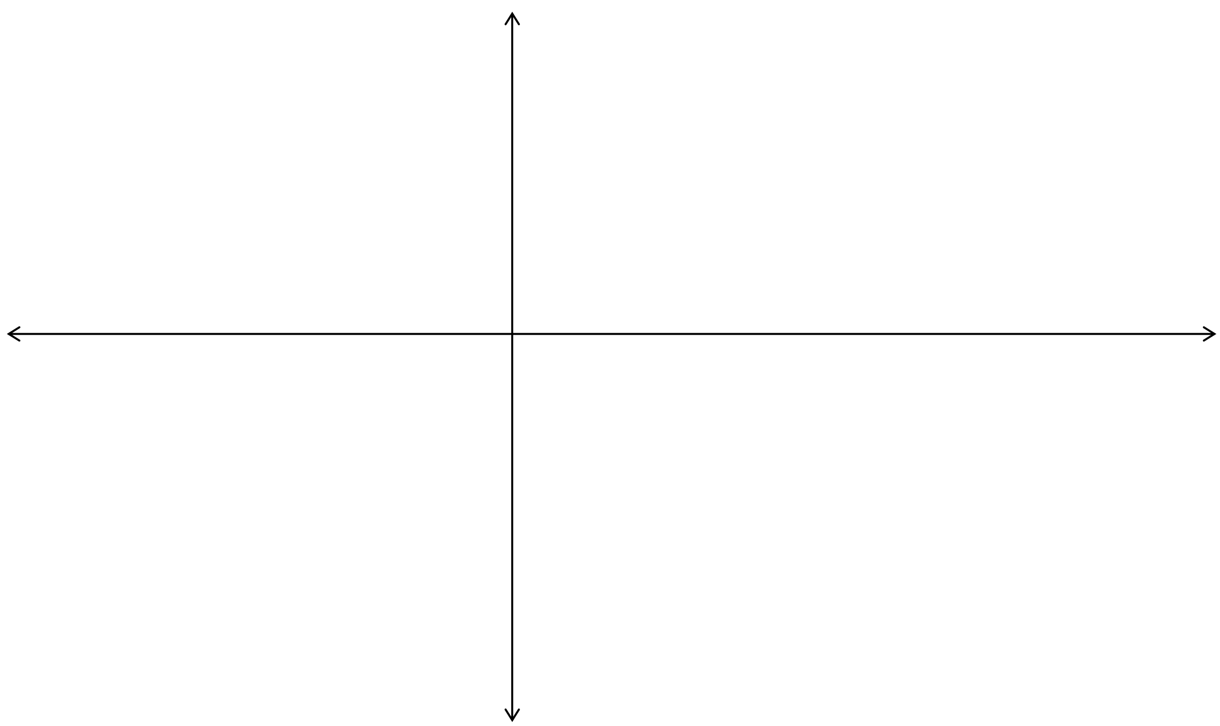
Do not work with anyone on this, but you may refer to your notes and text, and even the videos.

1. For the following factored polynomial, give each of the following data and then sketch the graph:

$$P(x) = -(2x+1)^2(x-3)$$

Degree______ Leading coefficient _____ y-intercept ____________

Roots & multiplicities: r_1 = _____ M_1 = _____; r_2 = _____ M_2 = _____



2. Completely factor the polynomial

$$f(x) = x^5 - 5x^4 + 3x^3 + 13x^2 - 8x - 12 = 0$$

First part of solution: The fundamental theorem of algebra tells us:

If r is a root of polynomial $f(x)$, then $(x - r)$ is a factor, and so $\dfrac{f(x)}{(x-r)}$ is another factor.

By the rational root theorem, we know 1 and –1 are always *possible* rational roots. They are easy to test, so we test them:

$x = 1$: $f(1) = 1 - 5 + 3 + 13 - 8 - 12 = -8 \neq 0$. So $x - 1$ is not a factor.

$x = -1$: $f(-1) = (-1)^5 - 5(-1)^4 + 3(-1)^3 + 13(-1)^2 - 8(-1) - 12 = 0$. $x - (-1) = x + 1$ is a factor.

Hence, $\dfrac{f(x)}{(x+1)}$ gives another factor, $x^4 - 6x^3 + 9x^2 + 4x - 12$. (Verify for yourself, for practice.)

It's possible $x = -1$ is *also* a root of this polynomial (and so its multiplicity would be greater than one). We try it:

$$f(-1) = (-1)^4 - 6(-1)^3 + 9(-1)^2 + 4(-1) - 12 = 16 - 16 = 0$$

$x + 1$ is again a factor. Long division yields another polynomial factor:

$$\frac{x^4 - 6x^3 + 9x^2 + 4x - 12}{x+1} = x^3 - 7x^2 + 16x - 12$$ (Verify for yourself, for practice.)

So far the factorization is $(x+1)^2(x^3 - 7x^2 + 16x - 12)$. The polynomial above evaluated at $x = -1$ is *not* zero. So we're done with that root.

The list of all possible rational roots relies only on the constant term, –12, since the leading coefficient is 1. Thus, the next smallest possible rational roots are $x = 2$ and –2.

Substituting $x = 2$ into $x^3 - 7x^2 + 16x - 12$ gives $(2)^3 - 7(2)^2 + 16(2) - 12 = 0$. So $x - 2$ is a factor.

One last long division $\dfrac{x^3 - 7x^2 + 16x - 12}{x-2} = x^2 - 5x + 6$. This factors easily as $(x - 3)(x - 2)$.

The complete factorization is: $f(x) = (x+1)(x+1)(x-2)(x-2)(x-3) = (x+1)^2(x-2)^2(x-3)$.

The roots are –1, 2 and 3. Their multiplicities are even, even and odd. So the graph will 'bounce' at the first two and cross at the third.

Now fill in the items below, referencing the function and the roots found above, and their multiplicities, we see we can now graph this fifth degree polynomial by the usual features:

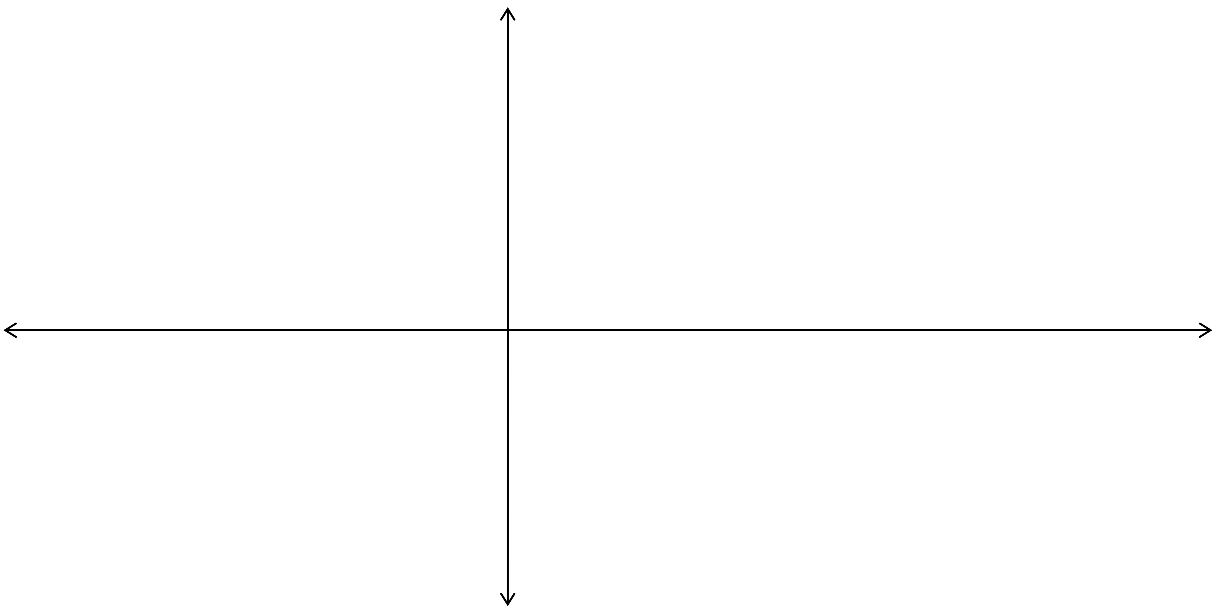
End behavior:

y-intercept:

Roots (*x*-intercepts):

Multiplicities:

Graph it here.



3. $f(x) = -x^4 + 3x^3 + 9x^2 - 23x + 12$

a) Degree:

b) Leading coefficient:

c) Use arrows to describe the end behavior of the graph of *f*.

d) *y*-intercept:

e) Show $x = 1$ is a root of *f*:

f) Use the fact that $x = 1$ is a root to help completely factor $f(x)$. Thereby determine all x-intercepts.

g) Graph f: