Math 224: Exam 1 **Name** ___________________________________

Spring 2016 **Instructor** ________________________________

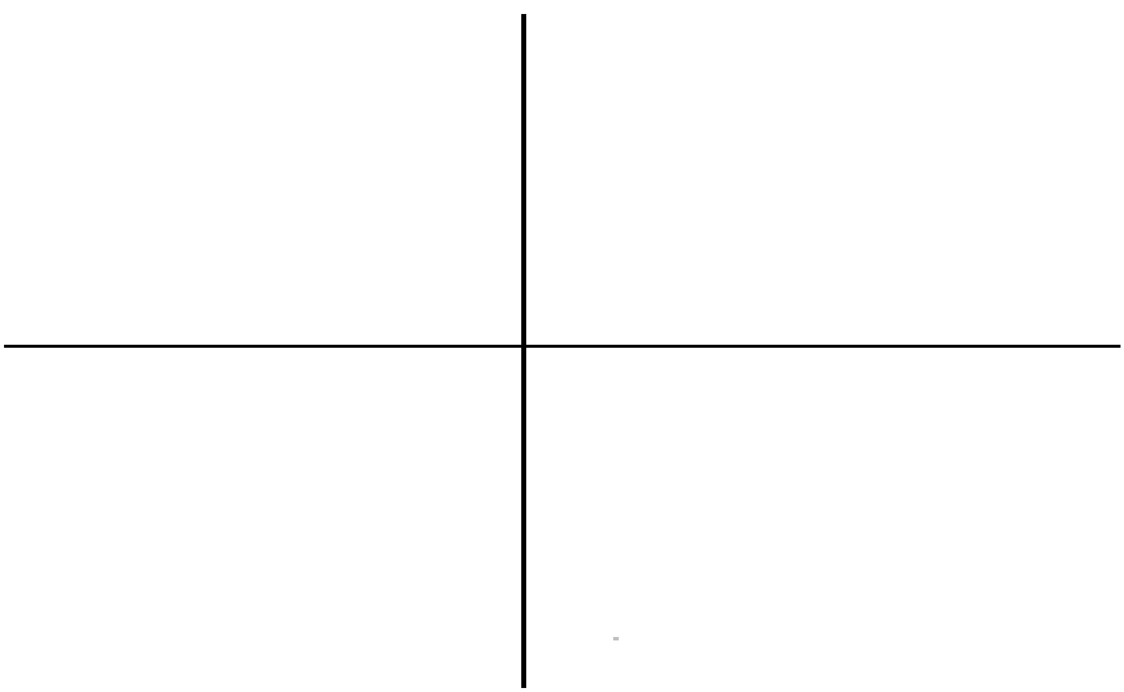
Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**	**Course Points**
Points	**12**	**8**	**20**	**11**	**15**	**10**	**12**	**12**	**100**	**300**
Score										

- Calculators and/or other electronic devices are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- There are problems on both front and back of the pages.
- If you need scrap paper, ask your instructor. You may not use your own. If you do use scrap paper, make sure to hand it in at the end of the exam. We will not read work on scrap paper unless you indicate for us to do so.
- It is highly advised that you complete this exam in pencil rather than pen.

1. (12 points) Sketch the graph of a function f with domain (-3, 4] satisfying all six of the following criteria.

a) $\lim_{x \to -3^+} f(x) = \infty$ b) $\lim_{x \to 0} f(x) = -1$ c) $f(0) = 1$

d) $\lim_{x \to 2} f(x) = -3$ e) $f'(2)$ does not exist. f) $f(4) = 0$



2. (8 points) Suppose $f(x) = \frac{2x+8}{x^2+2x-8}$.

Find the equations of all vertical asymptotes to the graph of f. Use appropriate limits to justify your answer.

3. (20 points) Evaluate each of the following limits, if they exist. If your answer is "DNE," be sure to fully explain why.

a) $\lim_{x \to \frac{1}{2}} \frac{\sqrt{2x+3}-2}{2x-1}$

b) $\lim_{x \to 4^-} \frac{|x-4|}{3x^2-3x-36}$

c) $\lim_{x \to 0} \frac{x+\sin(2x)}{x}$

d) $\lim_{x \to 3} \frac{x^2+6x+5}{x^2+2x-15}$

4. (11 points) Suppose $f(x) = \frac{3}{x}$

a) Show that $f'(x) = -\frac{3}{x^2}$ using the limit definition of derivative. No credit will be awarded for any other method.

b) Find the equation of the line tangent to f at the point where $x = 6$. Any form of linear equation is acceptable.

5. (15 points) Find the indicated derivatives for each expression. You may use differentiation shortcuts. You do not need to simplify your answers.

a) $\frac{d}{dx}[(2x^2 - 3)^3(5 - 2x)^4]$

b) $\frac{d}{dx}\left[\frac{\sin^2 x}{\cos(x) + 1}\right]$

c) $\frac{d}{dx}\left[\tan^3\left(\frac{2}{x}\right)\right]$

6. (10 points) Suppose $s(t) = 2t^2 + 3$ gives the position, in inches, of an object after t seconds, for all $t \geq 0$.

a) Find the average velocity of the object over the interval from $t = 1$ to $t = 7$. Express your answer with appropriate units.

b) Find the instantaneous velocity of the object at $t = 3$.

7. (12 points) Suppose $f(x) = \left\{\begin{array}{cc} \frac{1}{x} & \text{if } x < -1 \\ \cos(\pi x) & \text{if } -1 \leq x < 2 \\ \sqrt{x-1} & \text{if } 2 < x \leq 5 \\ x^2 - 4x & \text{if } x > 5 \end{array}\right\}$

a) Is f continuous at $x = -1$? Justify your answer completely.

b) Is f continuous at $x = 2$? Justify your answer completely.

c) Is f continuous at $x = 5$? Justify your answer completely.

8. (12 points) Suppose f and g are both differentiable functions for their entire domains. Function values and values of their derivatives when $x = 5$ and $x = -3$ are given in the following table:

x	f	g	f'	g'
5	2	-3	4	6
-3	8	0	-2	1

Using the above information, determine the following values, if they exist. If a value does not exist, explain why. Show your work.

a) $(f + g)'(-3)$

b) $(fg)'(5)$

c) $\left(\frac{f}{g}\right)'(-3)$

d) $(f \circ g)'(5)$