Exam 4 (previous semester) My Key

Name ______________________________

1. (4 points) Determine two coterminal angles (one positive and one negative) for each angle below.

a) -130° 230°, -490

b) $\frac{11\pi}{6}$ $\frac{23\pi}{6}$, $-\frac{\pi}{6}$

2. (4 points) Find the complement and supplement for each angle.

a) 57°

Complement = 33

Supplement = 123

b) $\frac{2\pi}{5}$ $\frac{5}{10}$

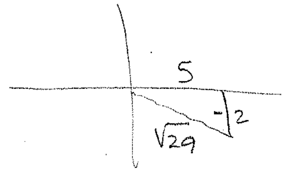
Complement = $\frac{3\pi}{10}$

Supplement = $\frac{3\pi}{5}$

3. (4 points) Change the units of measure.

a) Change 115° to radians. $\frac{115}{180} \cdot \pi = \frac{23}{36}\pi$

b) Change 4^r to degrees. $4 \cdot \frac{180}{\pi} = \frac{720}{\pi}$ r



4. (16 points) Angle θ (in standard position) has terminal side that goes through $(5, -2)$.

a) What are the six trigonometric values for θ?

$\sin(\theta) =$ $-2/\sqrt{29}$

$\cos(\theta) =$ $5/\sqrt{29}$

$\tan(\theta) =$ $-2/5$

$\sec(\theta) =$ $\sqrt{29}/5$

$\csc(\theta) =$ $-\sqrt{29}/2$

$\cot(\theta) =$ $-5/2$

b) What are the coordinates of the point where the terminal side of θ intersects the unit circle?

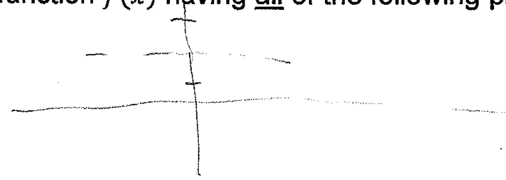
$\left(\frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}}\right)$

5. (8 points) Give the equation of a cosine function $f(x)$ having all of the following properties:

- Its range is $[1, 5]$
- Its period is 8π
- It has been reflected through the x-axis

$8\pi = \frac{2\pi}{B}$

$B = 1/4$



$f(x) =$ $2\cos\left(\frac{-x}{4}\right)$

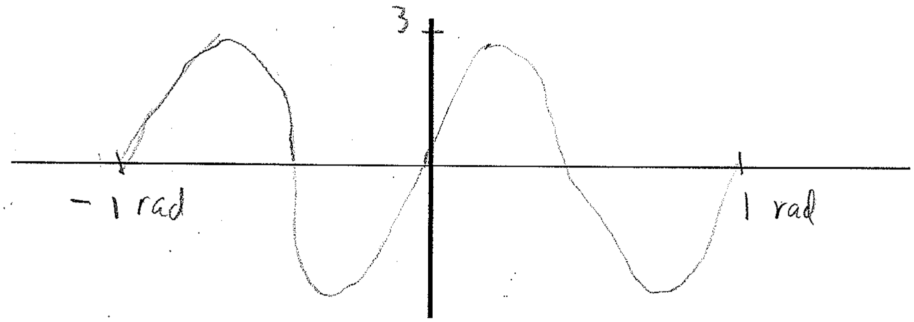
6. (8 points) Given $f(x) = 3\sin(2\pi x)$

$\frac{2\pi}{B} = 2\pi$, $B = 1$

a) What is the amplitude of f? 3

b) What is the period of f? 1

c) Sketch the graph of f over at least two full periods on the axes below.



7. (16 points) Evaluate each of the following:

$\cos(\pi) =$ -1

$\cos\left(\frac{11\pi}{6}\right) =$ $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\left(\frac{\pi}{3}\right) =$ $\frac{\sqrt{3}}{2}$

$\sin\left(\frac{5\pi}{4}\right) =$ $-\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

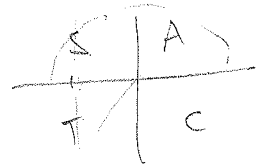
$\tan\left(\frac{8\pi}{3}\right) =$ $-\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\sin\left(\frac{5\pi}{12}\right) =$ $\sin\frac{\pi}{12} = \sin\left(\frac{\pi/6}{2}\right) = +\sqrt{\frac{1-\cos\pi/6}{2}}$

$= \sqrt{\frac{1-\sqrt{3}/2}{2}}$

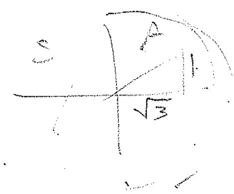
8. (15 points) Evaluate each of the following:

$\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$



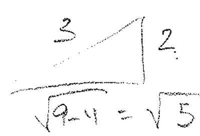
$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$ $\frac{\pi}{6}$

$\frac{1}{\sqrt{3}}$

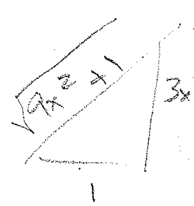


$\arcsin\left(\sin\left(\frac{3\pi}{2}\right)\right) =$ $= \arcsin\left(\sin\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{2}$

$\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right) =$ $\frac{\sqrt{5}}{3}$



$\sec(\arctan(3x)) =$ $\sqrt{9x^2+1}$



9. (8 points) Verify the trigonometric identity:

$$\frac{\sin(\theta)}{1+\cos(\theta)} + \frac{1+\cos(\theta)}{\sin(\theta)} = 2\csc(\theta)$$

$$\frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta\,(1+\cos\theta)} \stackrel{?}{=} 2\csc\theta$$

$$\frac{2+2\cos\theta}{\sin\theta\,(1+\cos\theta)} = \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}} = 2\csc\theta \checkmark$$

10. (8 points) Verify the trigonometric identity:

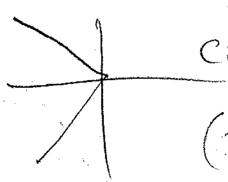
$$\frac{\cos(\alpha-\beta)}{\sin(\alpha)\sin(\beta)} = \cot(\alpha)\cot(\beta) + 1$$

$$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\sin\beta} = \left(\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}\right) + \frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}$$

$$\cot\alpha\cot\beta + 1 \checkmark$$

11. (8 points) Solve for x in the following equation. Find ALL solutions.

$2\cos(5x) = -\sqrt{2}$



$\cos 5x = -\frac{\sqrt{2}}{2}$

(2 sets of solns)

① $5x = \frac{3\pi}{4}, \frac{11\pi}{4}, \ldots$

$x = \frac{3\pi}{20}, \frac{11\pi}{20}, \ldots$

② $5x = \frac{5\pi}{4}, \frac{13\pi}{4}, \ldots$

$x = \frac{5\pi}{20}, \frac{13\pi}{20}, \ldots$

12. (8 points) Solve for x in the following equation. Find all solutions on the interval $[0, 2\pi)$.

$1 + \sin(x) = 2\cos^2(x)$

$1 + \sin x = 2(1 - \sin^2 x)$

$1 + \sin x - 2 + 2\sin^2 x = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

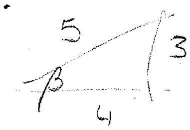
$x = \frac{\pi}{6}, \frac{5\pi}{6}$

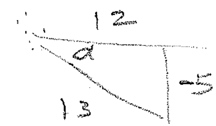
&

$x = \frac{3\pi}{2}$

13. (20 points) Suppose $\sin(\alpha) = -\frac{5}{13}$ and $\cos(\beta) = \frac{4}{5}$ such that $\frac{3\pi}{2} < \alpha < 2\pi$ and $\frac{3\pi}{2} < \beta < 2\pi$

a) Find $\sin(\beta)$. $\boxed{3/5}$





b) Find $\cos(\alpha)$. $\boxed{12/13}$

c) Find $\cos(\alpha+\beta)$.

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \frac{12}{13}\cdot\frac{3}{5} - \frac{-5}{13}\cdot\frac{3}{5} = \boxed{\frac{51}{65}} > 0$$

d) What quadrant is $\alpha+\beta$ in? Justify your answer. $\cos(\alpha+\beta) > 0$, so in either I or IV

$$\frac{3\pi}{2} + \frac{3\pi}{2} < \alpha + \beta < 2\pi + 2\pi \longrightarrow 3\pi < \alpha+\beta < 4\pi$$

e) Find $\sin(2\alpha)$.

$$2\sin\alpha\cos\alpha = 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \frac{-120}{169} < 0$$, so either III or IV

f) What quadrant is 2α in? Justify your answer.

$$\frac{3\pi}{2} < \alpha < 2\pi \longrightarrow 3\pi < 2\alpha < 4\pi$$ same

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 > 0$$ Q IV

g) Find $\cos\left(\frac{\beta}{2}\right)$.

$$= \sqrt{\frac{1+\cos\beta}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{9/5}{2}} = \boxed{\sqrt{9/10}} \text{ or } 3/\sqrt{10}$$

h) What quadrant is $\frac{\beta}{2}$ in? Justify your answer.

$$\frac{3\pi}{2} < \beta < 2\pi \longrightarrow \frac{3\pi}{4} < \frac{\beta}{2} < \pi$$ $\boxed{\text{Q II}}$