

Memorize the definition of logarithm:

" $\log_a x = y$ means $a^y = x$ "
The log. of x is y "

Ex. $\log_{(2)} 4 = y$ means $2^y = 4$, $y = 2$

Ex. $\log_{10} 100 = y$ means $10^y = 100$, $y = 2$
not written as a norm

We don't write $\log_{10} x$, but when we
see $\log x$, we mean $\log_{10} x$.

10 is called the base of the common logarithm

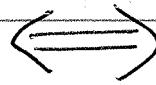
So " $\log x$ " means "common log of x "
means " $\log_{10} x$ ".

Memorize all the log properties.

$\log_a x = y$ ← logarithm
base a \nwarrow antilog
 \uparrow exponent

means we raise a to the y power to get x .

$$a^x = a^y \text{ iff } x = y$$



"Like bases raised to

Ex $2^x = 2^y \Leftrightarrow x = y$

Ex $2^x = 4^y \Leftrightarrow 2^x = 2^{2y}$

$$2^x = (2^2)^y \Leftrightarrow x = 2y$$

Ex Find x given that

$$3^x = \left(\frac{1}{3}\right)^{2x-1}$$

First, turn $\frac{1}{3}$ into 3 , i.e.

rewrite as $3^x : \left|\frac{1}{3} = 3^{-1}\right|$

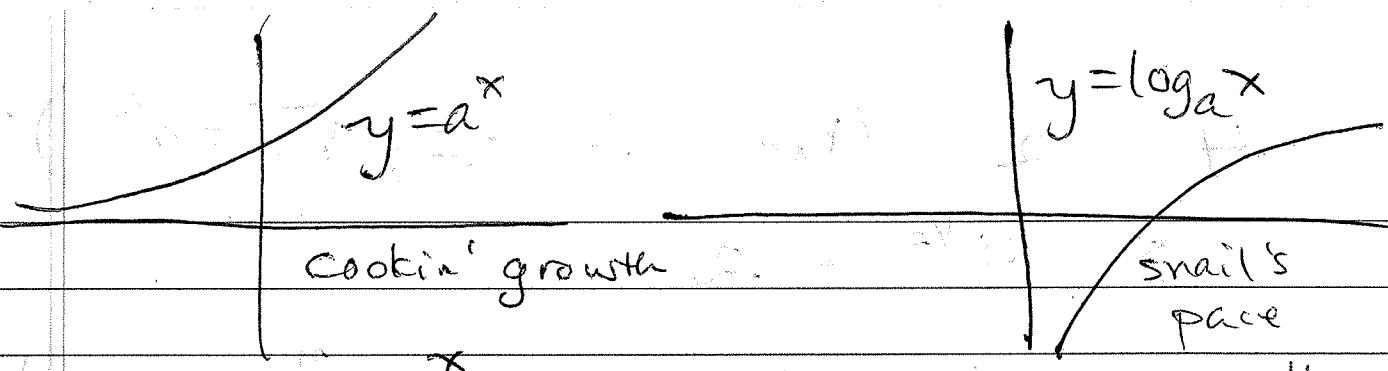
Now, $3^x = (3^{-1})^{2x-1}$

$$3^x = 3^{1-2x}$$

so by the prop $a^x = a^y \Leftrightarrow x = y$

$$x = 1 - 2x$$

$$\begin{array}{l} 3x = 1 \\ x = \frac{1}{3} \end{array}$$



$$f(x) = a^x$$

$$g(x) = \log_a x$$

$f + g$ are mutual inverses

11/5 Find the following

$$\log x = 10.00$$

Evaluate using the def of $\log_a x = y$ means $a^y = x$

$$y = \ln 1 \iff e^y = 1 \iff y = 0$$

$$y = \log_4 2 \iff 4^y = 2 \iff y = \frac{1}{2}$$

$$y = \log(0.01) \iff 10^y = 0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2} \iff y = -2$$

$$y = \log_3 \frac{1}{27} \iff 3^y = \frac{1}{27} = \frac{1}{3^3} = 3^{-3} \iff y = -3$$

$$y = \log_{\frac{1}{2}} 8 \iff \left(\frac{1}{2}\right)^y = 8 \iff 2^{-y} = 8$$

$$y = (\log_4 2 +) \log_4 16 = \frac{1}{2} + 2 = 2\frac{1}{2}$$

$$\rightarrow y = 2 \ln e^3 + \ln e^{-5} + e^{\ln 3}$$

$$4^{\frac{1}{2}} = 2 \quad (\text{know } \sqrt{4} = 2; \sqrt{a} = a^{\frac{1}{2}})$$

$$\text{So } 4^{\frac{1}{2}} = 2$$

$$\log_3 27 = y \leftrightarrow 3^y = 27 \leftrightarrow y = 3$$

$$\log (.001) = y \leftrightarrow 10^y = .001 \leftrightarrow$$

$$10^y = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3} \rightarrow y = -3$$

$$\log 1,000,000 = y \leftrightarrow 10^y = 1,000,000$$

$$\rightarrow y = 6$$

$$\log .000001 = y \leftrightarrow 10^y = .000001$$

$$10^y = \frac{1}{10^6} = 10^{-6}$$

$$y = -6$$

$$\log_2 \frac{1}{64} = y \leftrightarrow 2^y = \frac{1}{64} = \frac{1}{2^6} = 2^{-6}$$

$$y = -6$$

$$y = \log_{\frac{1}{2}} 8 \Leftrightarrow \frac{1}{2}^y = 8$$

$$\Leftrightarrow 2^{-y} = 8 \Leftrightarrow 2^{-(-3)} = 8$$

i.e. $2^3 = 8$

$$\Leftrightarrow y = -3$$

Cheat: $y = \log_{\frac{1}{2}} 8 \Leftrightarrow -3 = \log_2 8$

Then $(\frac{1}{2})^{-3} = 2^3 = 8 \checkmark$

$$\ln x \equiv \log_e x \text{ where } e \approx 2.718$$

"e" is the base that relates to many natural growth + decay processes!!

All the usual rules of $\log_b x = y$ apply.

Main rule: $\log_b b = y \Leftrightarrow b^y = b \Leftrightarrow y=1$

So $\log 10 = 1$ and $\ln e = 1$

$$10^1 = 10$$

$$e^1 = e$$

Combine ~~the~~ properties to simplify log ~~expressions~~ expressions.

$$\log 10^3 = 3 \log 10 = 3 \cdot 1 = 3$$

from $\log x^r = r \log x$

$$\log 7000 = \log(7)(1000) = \log 7 + \log 1000$$

$$\log 7000 = \log 7 + 3 = 3 + \log 7$$

$$2 = [1.21 + 3 < 4]$$

Relative Magnitude of Logs:

How big is? $\log 80 < \log 100 = 2$

$$2 = \log 10 < \log 80$$

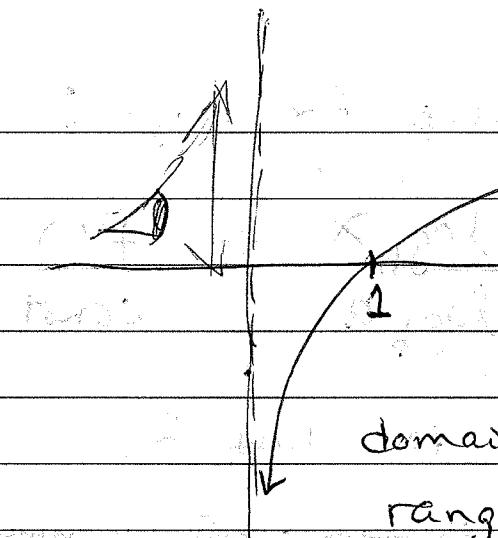
$$80 \quad 1 < \log 80 < 2$$

Another approach: $\log 80 = \log(8)(10)$

$$\log(8)(10) = \log 8 + \log 10 = \log 8 + 1$$

$$\log 8 + 1 < 2$$

Mother fun $y = \ln x$



root

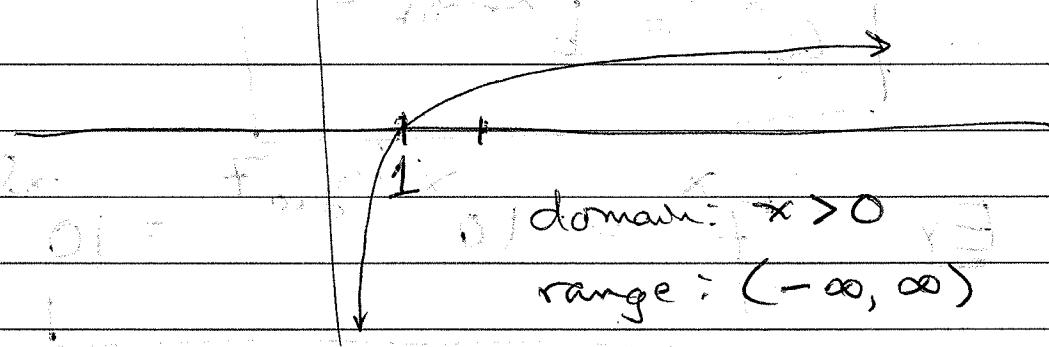
$$y = 0 = \ln x \\ b/c e^0 = 1$$

domain: $x > 0$

range: $(-\infty, \infty)$

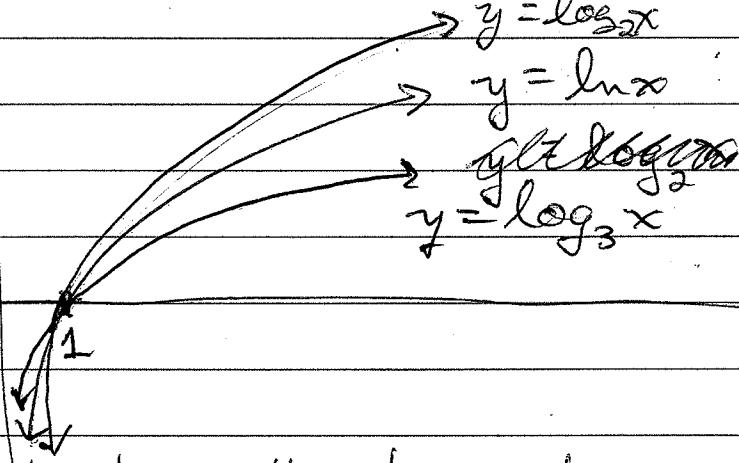
↳ Goes to $+\infty$ very slowly!

$y = \log x$ rises much slower than $\ln x$



On same graph, show $y = \log_2 x$ + $y = \ln x$

$e \approx 2.718$



The greater the base, the slower the rise of log fun.
Because, as inverses of exponential funs, they behave in opposite manner.

Change of base formulae for logs :

$$\log_a x = \frac{\log_b x}{\log_b a} + f(x)$$

const

From base a to base b

Ex : Write $\log_7 x$ in terms of common logs:

$$\log_7 x = \log_{10} x / \log_{10} 7 = [\log x / \log 7]$$

Change of formula for exponentials :

base

$$a^x = b^{x \log_b a}$$

$$\text{Ex } 7^x = 10^{x \log_{10} 7} = 10^{x \log 7}$$

Ex

Sometimes we need to use several properties of exponentials to solve a problem (equation)

$$2^{x+1} = 4^{x-3}$$

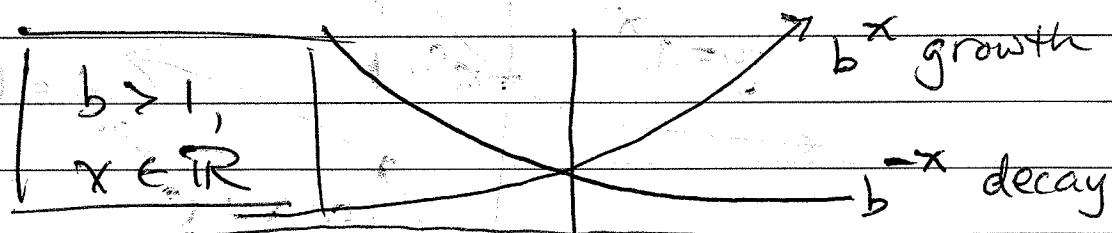
Can we rewrite one base to look like the other, using the def of exp?

Yes $2^{x+1} = (2)^{x-3}$

$$\underline{2^{x+1}} = \underline{2^{x-6}} \rightarrow x+1 = 2x-6$$
$$\underline{\quad\quad\quad} \rightarrow \underline{x = 7}$$

Like bases

The reason we do not entertain bases < 1 is seen in the graph of typical, useful exponential fn.



The nature of the exponential fn. is as ~~the~~ a picture of growth or decay.

(BTW)
 $y = 1^x = 1$

We see in nature this idea
of exponential growth or decay

$$y = a^x$$

We also see in nature the idea
of logarithmic growth.

$$y = \log_a x$$

These funcs are mutual inverses

$$y = a^x$$

$y = x$ (not an
asymptote)

$$a^0 = 1$$

$$y = \log_a x$$

$$\log_a x = 0$$

$$x = 1$$