

## Math 220 - Calculus f. Business and Mgmt - Worksheets 10, 11

### Worksheet 10 - 11 - Limits as $x$ Approaches a

#### Basic limits

Exercise 1:

$$1a: \lim_{x \rightarrow 3} x^2 + 2x - 5, \quad 1b: \lim_{x \rightarrow 3} \sqrt{x^2 + 16}, \quad 1c: \lim_{x \rightarrow 1} \ln(x), \quad 1d: \lim_{x \rightarrow 0} e^x$$

#### Simple Fractions

Exercise 2:

$$2a: \lim_{x \rightarrow 2} \frac{x^3 + 2x - 3}{7}, \quad 2b: \lim_{x \rightarrow 3} \frac{(x-2)(x-3)}{(x-3)}, \quad 2c: \lim_{x \rightarrow 1} \frac{3x^5 - 7x + 2}{4x^2 + 6},$$

$$2d: \lim_{x \rightarrow -2} \frac{3x^3 + 6x^2}{x + 2}, \quad 2e: \lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2},$$

#### Composite Functions

Exercise 3:

$$3a: \lim_{x \rightarrow 4} e^{x+2}, \quad 3b: \lim_{x \rightarrow 4} \ln(x^2 + 5x + 2), \quad 3c: \lim_{x \rightarrow 8} \ln(x - 12),$$

$$3d: \lim_{x \rightarrow 2} \sqrt[3]{5x^2 + x + 5},$$

#### Piecewise Functions

Exercise 4:

$$4a: f(x) = \begin{cases} 2x + 3 & \text{if } x < 3, \\ x^2 & \text{if } x \geq 3. \end{cases} \quad \text{Evaluate limits as } x \text{ approaches } -5, 3 \text{ and } 6.$$
$$4b: f(x) = \begin{cases} 4x & \text{if } x < 2, \\ x + 3 & \text{if } x \geq 2. \end{cases} \quad \text{Evaluate limits as } x \text{ approaches } 0, 2 \text{ and } 5.$$
$$4c: f(x) = \begin{cases} 3x + 1 & \text{if } x < 1, \\ x + 3 & \text{if } x > 1. \end{cases} \quad \text{Evaluate limits as } x \text{ approaches } -3, 1 \text{ and } 4.$$

Identify points of discontinuity and compare your findings to the answers to the same question on the worksheet for class 9.

#### Fractions of the form non-zero / zero

Exercise 5:

$$5a: \lim_{x \rightarrow 1^+} \frac{x^2}{x-1}, \quad 5b: \lim_{x \rightarrow -2^+} \frac{x^3 - 6x^2 + 2}{x+2}, \quad 5c: \lim_{x \rightarrow -3^+} \frac{x^3 + 6x}{x^2 + x - 6}, \quad 5d: \lim_{x \rightarrow 2^-} \frac{x^3 + 6x}{x^2 + x - 6}.$$

#### Vertical Asymptotes

Exercise 6:

6a: Give the equation(s) for any vertical asymptotes that exist for the fractions in the section above. Note that the last two problems deal with the same function with different limits)

6b: Find vertical asymptotes for      6b.1:  $\frac{3x^3 + 3x^2}{x^2 + 7x + 6}$ ,      6b.2:  $\frac{x^2 + 5x + 5}{x^2 - 2x - 8}$ ,