MATH 108-03 KEY Trig Quiz #1

1. After each item the quadrant of the terminal angle is given:

Solution:

$\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (QIV) $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ (QIII) $\tan\left(\frac{4\pi}{3}\right) = \sqrt{3}$ (QIII)

$\sin(5\pi) = 0$ $\cos\left(\frac{25\pi}{4}\right) = \frac{1}{\sqrt{2}}$ (QI) $\tan\left(-\frac{11}{2}\pi\right)$ undef.

(between QII&III) (between QI&II)

2. Given sin $\theta = -3/5$ and tan $\theta < 0$:

a) Find the x-coordinate of the terminal side of θ with y-coordinate corresponding to –6.

Solution: The triangle lies in QIV, known from given information sin $\theta = -3/5$ and tan $\theta < 0$.

So this is the 3-4-5 right triangle, scaled up to 6-8-10. Thus, the x-coordinate is 8.

b) What are the coordinates of the point where the terminal side of θ intersect the unit circle?

Solution: You could answer this in one of two methods. Since (6, –8) are on a circle of radius 10, dividing each coordinate by 10 scales them back to the point of intersection with the unit circle.

In detail, (8, –6) = (10 cosθ, 10 sin θ). Hence, (8/10, –6/10) = (cosθ, sin θ). Reducing gives (4/5, –3/5). Thus the x-coordinate is 4/5.

Similarly, we note that y = r sin θ for a circle of radius r, so y = 1 sin θ for the unit circle. It is given that sin $\theta = -3/5$, so the y = 1 sin θ = (1)(–3/5) = –3/5.

Likewise, from the information on sin $\theta = -3/5$ we get cos $\theta = 4/5$ in QIV.

3. Find the length of the arc that subtends an angle measuring 105° in a circle of radius 8 cm.

Solution: Arc length s = r θ = 8 x 105° x $\pi/180° = 14\pi/3$. The answer makes sense, since it is about 15 and the circumference of the circle is $2\pi r = 2\pi(8) = 16\pi$, about 50 cm. The angle that is subtended by 15/50 of the circle, that is, something less than 1/3, is credibly 105°.

4. Suppose $\theta = 2\pi/9$.

a) What is the complement of θ?

Solution: $\pi/2 - 2\pi/9 = 9\pi/18 - 4\pi/18 = 5\pi/18$

b) What is the supplement of θ?

Solution: $\pi - 2\pi/9 = 7\pi/9$

c) Name 2 angles co-terminal with θ, one negative and one positive.

Solution: $2\pi/9 + 2\pi = 20\pi/8$; $2\pi/9 - 2\pi = -16\pi/8$

d) What is the measure of θ in degrees?

Solution: $2\pi/9 \times 180/\pi = 40°$

5. Graph the following function, showing **one** full period of the graph with the endpoints and any intercepts labeled clearly:

Solution: $f(x) = -\sin\left(3x - \frac{\pi}{2}\right) = -\sin\left[3\left(x - \frac{\pi}{6}\right)\right]$

A = –1 so amp = abs value A = 1 (graph flips over x-axis)

B = 3, so period = $2\pi/B = 2\pi/3$

C = $\frac{\pi}{6}$ so the horizontal shift is $\frac{\pi}{6}$ to the right

Endpoints: Solve $3x - \frac{\pi}{2} = 0$ and $3x - \frac{\pi}{2} = 2\pi$ to get x = $\frac{\pi}{6}$ and x = $\frac{5\pi}{6}$

6. Graph the following function, also showing endpoints, but this time show **two** full periods:

$g(x) = 3\cos(2\pi x)$

Solution: $g(x) = 3\cos(2\pi x)$

A = 3 so amp = abs value A = 3 (graph stretches by a factor of 3)

B = 2π, so period = $2\pi/2\pi = 1$ radian

C = 0 so there's no horizontal shift

Endpoints: Solve $2\pi x = 0$ and $2\pi x = 2\pi$ to get x = 0 and x = 1 (consistent with no horizontal shift).