MATH 108 Quiz #4 Spring 2017 Name Key

Directions: Answer each question as completely as possible. Show all work for credit. Good luck.

1. Write the equation of a function $g(x)$ that is obtained by taking $f(x) = \sqrt{x}$ and performing the following transformations, in this order:

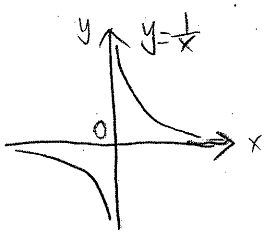
- a horizontal shift to the left by 2 units $\longrightarrow \sqrt{x+2}$
- a vertical compression by a factor of 3 $\frac{1}{3}\sqrt{x+2}$
- a vertical shift downward by 5 units $\frac{1}{3}\sqrt{x+2}-5$
- a reflection over the y-axis

$g(x) = \frac{1}{3}\sqrt{-x+2} - 5$

2. Suppose $h(x) = -\frac{1}{x-3} + 1$

Draw a rough sketch of the graph of h, performing transformations of $f(x) = \frac{1}{x}$ one at a time until h is obtained. Show a separate graph for each transformation, as done in class.

1



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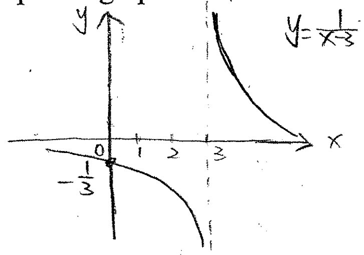

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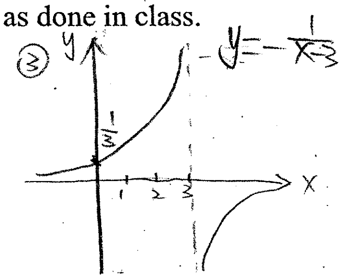

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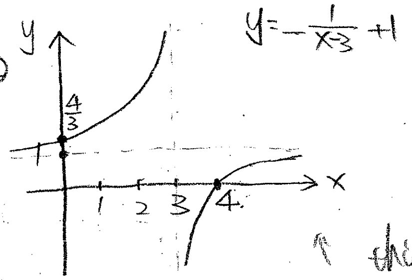


↑ this is the sketch of $h(x) = -\frac{1}{x-3}+1$

odd fn.

3. Suppose $f(x) = x^5 - 5x^4 + 3x^3 + 13x^2 - 8x - 12$

a) What is the degree of f? deg $f = 5$.

b) What is the leading coefficient of f? leading coefficient $f = 1$.

c) Use arrows to describe the end behavior of the graph of f.

f(x) graph falls to the left and rises to the right

d) Find the y-intercept of the graph of f.

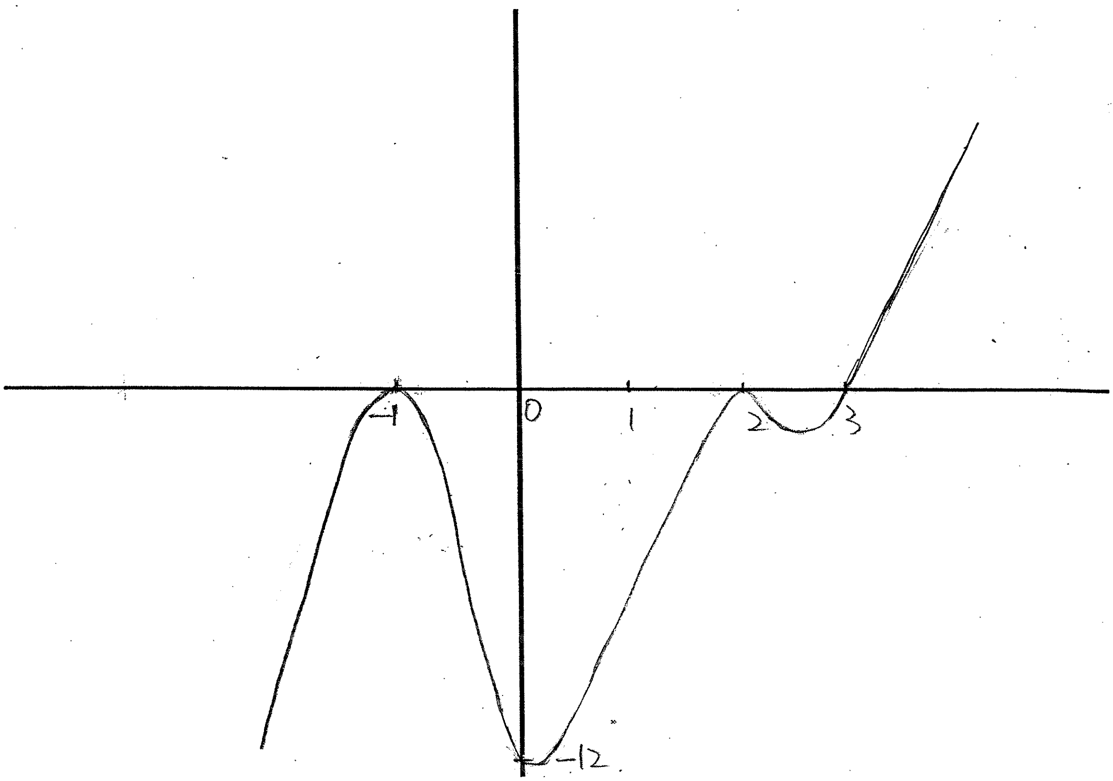
$f(0) = -12$

e) Write a complete factorization of f. Use this to determine any x-intercepts of the graph of f.

finial ans : $f(x) = (x+1)^2(x-2)^2(x-3)$.

see the solution in next page

f) Use the above information to draw a rough sketch of the graph of f. Your graph should have the correct intercepts and end behavior, and should have correct behavior at each x-intercept.



Solution:

- From the rational root thm, possible rational roots are
  $\{\pm\frac{1}{1}, \pm\frac{12}{1}, \pm\frac{6}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{3}{1}\}$
  that is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$.

- First try $x = 1, -1, 2, -2$, since they are the smallest possible rational roots. Evaluate $f(1)$, $f(-1)$, etc to see if you get a root.
  $f(1) \neq 0$. ← 1 can't be a root for our fn.
  $f(-1) = 0$ ← -1 is a root.

$$\begin{array}{r|rrrrrr} -1 & 1 & -5 & 3 & 13 & -8 & -12 \\ & & -1 & 6 & -9 & -4 & 12 \\ \hline & \underbrace{1 \quad -6 \quad 9 \quad 4 \quad -12}_{\text{these are our coefficients of the next factor.}} & & & & & 0 \end{array}$$

→ $y = x^4 - 6x^3 + 9x^2 + 4x - 12$.

So far, we have $(x+1)(\quad)$

Then, try $x=-1$ again in case we have a root with multiplicity >1.
we can see when $x=-1$, $y=0 \Rightarrow x=-1$ is a root of y.

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 9 & 4 & -12 \\ & & -1 & 7 & -16 & 12 \\ \hline & \underbrace{1 \quad -7 \quad 16 \quad -12}_{\text{these are our coeff. of the next factor.}} & & & & 0 \end{array}$$

→ $y_1 = x^3 - 7x^2 + 16x - 12$

So far, we have $(x+1)^2(\quad)$

Then, try $x=2$. When $x=2$, $y_1 = 0 \Rightarrow x=2$ is a root.

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 16 & -12 \\ & & 2 & -10 & 12 \\ \hline & \underbrace{1 \quad -5 \quad 6}_{\text{these are our coeff. of the next factor.}} & & & 0 \end{array}$$

$y_2 = x^2 - 5x + 6$.

So far, we have $(x+1)^2(x-2)(x^2-5x+6)$ and this is equal to $(x+1)^2(x-2)(x-2)(x+3)$.