Math 108 Practice for Exam 1

1. Demonstrate that the decimal $3.6\overline{12}$ is rational by rewriting it as an integer divided by another integer.

Let $x = 3.6\overline{12}$

$100x = 361.21212\ldots$

(Here, $10x = 36.1212\ldots$ doesn't line up to cancel $\overline{12}$)

$\longrightarrow$ Subtract

$$\begin{aligned} 100x &= 361.21\cancel{2}\cancel{12}\ldots \\ -\ x &= 3.6\cancel{12}\cancel{12}\ldots \\ \hline 99x &= 357.6 \end{aligned}$$

$x = \frac{357.6}{99} = \boxed{\frac{3576}{990}}$

2. Simplify each of the exponential expressions as completely as possible. Write all answers such that there are no negative exponents.

a) $\left(\frac{-2a^4b}{a^{-3}b^2}\right)^3 = \frac{(-2)^3(a^4)^3\,b^3}{(a^{-3})^3(b^2)^3} = \frac{-8\cdot a^{12}\cdot b^3}{a^{-9}\,b^6} = \frac{-8a^{12}a^9}{b^3}$

$= \boxed{\frac{-8a^{21}}{b^3}}$

b) $(2x^{-4}y^3)^3(3x^2y^{-2})^{-2} =$

$= 2^3x^{-12}y^3\cdot 3^{-2}x^{-4}y^4 = \boxed{\frac{8}{9}\cdot\frac{y^7}{x^{16}}}$

3. Simplify each of the radical expressions as completely as possible. Combine like terms where possible.

a) $\sqrt[3]{72a^4b^5} = \sqrt[3]{8\cdot 9\cdot a^3\cdot a^1\cdot b^3 b^2} = \boxed{2ab\sqrt[3]{9ab^2}}$

$8\cdot 9$

b) $9a\sqrt{20a^3b^2} + 7b\sqrt{45a^5} = 9\cdot 2\cdot a^2\cdot b\sqrt{5a} + 7\cdot 3\cdot a^2b\sqrt{5a}$

$4\cdot 5$ $\quad 9\cdot 5$

$a^2\cdot a$ $\quad a^4\cdot a$

b^2

$= 18a^2b\sqrt{5a} + 21a^2b\sqrt{5a}$

$= \boxed{39a^2b\sqrt{5a}}$

1. Multiply each of the following expressions. Combine like terms where possible.

a) $(2x^2 + 3x - 5)(x^2 - 7x + 2) =$

$$\begin{array}{r} 2x^2 + 3x - 5 \\ \cdot\ x^2 - 7x + 2 \\ \hline 2x^4 + 3x^3 - 5x^2 \qquad\qquad \\ -14x^3 - 21x^2 + 35x \qquad \\ + 4x^2 + 6x - 10 \end{array}$$

$$\boxed{2x^4 - 11x^3 - 22x^2 + 41x - 10}$$

b) $(\sqrt{x} - y)(\sqrt{x} + y)$

$= \sqrt{x}\sqrt{x} + \cancel{\sqrt{x}y} - \cancel{\sqrt{x}y} - y \cdot y$

$= x - y^2 = \boxed{x - y^2}$

2. Factor completely.

a) $12x^3y - 22x^2y + 8x^1y$

$= 2xy(6x^2 - 11x + 4)$

$= 2xy(6x^2 - 8x - 3x + 4)$

~~12·8 = 96~~

$6 \cdot 4 = 24$

$-8 \quad -3$

$\boxed{2xy(3x-4)(2x-1)}$

* b) $(4x^3 + 12x^2) - (9x + 27)$

$= 4x^2(x+3) - 9(x+3)$

$= (x+3)(4x^2 - 9) = \boxed{(x+3)(2x+3)(2x-3)}$

c) $125x^6 + 27y^3$

$= \boxed{(5x^2 + 3y)(25x^4 - 15x^2y + 9y^2)}$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 = 125x^6 \qquad b^3 = 27y^3$

$a = 5x^2 \qquad b = 3y$

$a^2 = 25x^4 \qquad b^2 = 9y^2$

KEY

1. Use process of complete the square to rewrite

$-5x^2 - 60x + 7$ as form $a(x-h)^2 + k$

① $-5x^2 - 60x \qquad + 7$ space!

② $-5(x^2 + 12x \qquad) + 7$ factor first 2 terms

③ $-5(x^2 + 12x + 36) + 7 + ?$ Complete the square $(\frac{1}{2} \text{ of } 12)^2$

④ Adjust constant +7 so original expression is not changed; we actually have $(-5)(36)$ "added" (i.e., subtracted $(5)(36)$) so we have to add it back:

$-5(x^2 + 12x + 36) + 7 + 5(36)$

⑤ $\boxed{-5(x+6)^2 + 187}$ Desired form, where $a = -5$, $h = -6$, $k = 187$

2.

$$
\begin{array}{r}
2x^2 - 6x + 3 \\
x^2 - x + 5 \enclose{longdiv}{2x^4 - 8x^3 + 19x^2 - 33x + 15} \\
-[2x^4 - 2x^3 + 10x^2] \\
\hline
-6x^3 + 9x^2 - 33x \\
-[-6x^3 + 6x^2 - 30x] \\
\hline
3x^2 - 3x + 15 \\
-[3x^2 - 3x + 15] \\
\hline
0
\end{array}
$$

*(d) at bottom: It comes down to algebra when you compare $f(-x)$ to $f(x)$ for even, $-f(x)$ for odd.

$f(-x) = \frac{-7x-1}{-x+3}$; $-f(x) = -\left(\frac{7x-1}{x+3}\right) = \frac{-7x+1}{x+3} \neq f(-x)$ (not odd)

$\neq f(x)$ (not even)

3. Suppose $f(x) = \frac{7x-1}{x+3}$

a) Use the definition of one-to-one to demonstrate that f is a one-to-one function.

(1) Suppose $f(a) = f(b)$

(2) So $\frac{7a-1}{a+3} = \frac{7b-1}{b+3}$

(3) Cross-multiply to combine like terms:

$(7a-1)(b+3) = (a+3)(7b-1)$

$\cancel{7ab}+21a-b-\cancel{3} = \cancel{7ab}-a+21b-\cancel{3}$

$21a-b = -a+21b$

$\cancel{22}a = \cancel{22}b$

(4) $\boxed{a=b}$ So $f(x)$ is 1-1.

b) Find f^{-1}, the inverse function of f.

(1) Write $f(x)$ as y:

$y = \frac{7x-1}{x+3}$

(2) Switch x & y: $x = \frac{7y-1}{y+3}$

(3) Solve for y:

$xy + 3x = 7y-1$

$xy - 7y = -3x-1$

$y(x-7) = -3x-1$

$y = \frac{-3x-1}{x-7} = f^{-1}(x)$

(4) Rename

c) Find the range of f.

Since Dom of f^{-1} = Range f, and it's clear $D_{f^{-1}}$ is all $x \neq 7$

then R_f is all $y \neq 7$, $(-\infty,7)\cup(7,\infty)$

d) Demonstrate that f is neither an odd function nor an even function. (There are many possible ways to do this.)

(1) Write $f(-x)$: $f(-x) = \frac{7(-x)-1}{-x+3}$

(2) Simplify: $f(-x) = \frac{-7x-1}{-x+3}$

(3) Compare to $f(x)$: $\frac{-7x-1}{-x+3} \neq \frac{7x-1}{x+3}$

$f(-x) \neq f(x)$

so f is ~~~~ not even

(Why not? Factor out (-1) in $f(-x)$)

(4) Compare to $-f(x)$:

$-\left(\frac{7x-1}{x+3}\right) = \frac{-7x+1}{x+3} \neq f(-x)$

So $f(x)$ is not odd.

3. Solve each of the following quadratic equations for x.

a) $2x^2 - 9x = -9$ $\longrightarrow$ $2x^2 - 9x + 9 = 0$

$(2x - 3)(x - 3) = 0$

$2x - 3 = 0$ $\quad$ $x - 3 = 0$

$\boxed{x = 3/2}$ $\quad$ $\boxed{x = 3}$

b) $(3x - 7)^2 = 71$

$\sqrt{(3x-7)^2} = \pm\sqrt{71}$

$3x - 7 = \pm\sqrt{71}$

$3x = 7 \pm \sqrt{71}$

$\boxed{x = \dfrac{7 \pm \sqrt{71}}{3}}$

c) $x^2 - 2x = 4$

$x^2 - 2x - 4 = 0$ Does not factor. Use QF.

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)} = \dfrac{2 \pm \sqrt{4 + 16}}{2}$

$= \dfrac{2 \pm \sqrt{20}}{2} = \dfrac{2 \pm 2\sqrt{5}}{2}$

$= \boxed{1 \pm \sqrt{5}}$

d) $4x^2 + 7x - 9 = 0$

Try "magic" or "ac" method

$(4)(-9) = -36$

There are no factors of (-36) which add up to 7 (of $7x$), so we need QF.

$x = \dfrac{-7 \pm \sqrt{49 - 4(4)(-9)}}{2(4)} = \dfrac{-7 \pm \sqrt{49 + 144}}{8} = \boxed{\dfrac{-7 \pm \sqrt{193}}{8}}$

1. Find the domain of each of the following functions.

a) $f(x) = \frac{x^2-9x-36}{2x^2+7x+3}$ ← only this matters

$Dom(f) =$ $2x^2+7x+3 \neq 0$

$(2x+1)(x+3) = 0$

$\boxed{x \neq -\frac{1}{2}, -3}$

$\boxed{(-\infty,-3)\cup(-3,-\frac{1}{2})\cup(-\frac{1}{2},\infty)}$

b) $g(x) = \sqrt[3]{x-5}$ $x-5$ = all reals, so x = all reals $(-\infty,\infty)$

$Dom(g) =$ $(-\infty,\infty)$

c) $h(x) = \begin{cases} x^2 & \text{if x is an even integer} \\ x-1 & \text{if x is an odd integer} \end{cases}$ This is domain

$Dom(h) =$ $\mathbb{Z}$ set of integers $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$

2. Suppose $f(x) = \frac{1}{x^4-9}$ and $g(x) = \sqrt{x-1}$.

a) Find $(f \circ g)(x)$. $= f(g(x)) = f(\sqrt{x-1}) = \frac{1}{(\sqrt{x-1})^4-9}$

$$= \frac{1}{(x-1)^2-9}$$

b) Find the domain of $f \circ g$.

First, look at D_g, ~~sa~~ since $g(x)$ is the input for $f(g(x))$.

D_g: $x-1 \geq 0$, or $x \geq 1$ $\boxed{[1,\infty)}$: D_g

Now, look at fcn $f(g(x)) = \frac{1}{(x-1)^2-9}$. What is its dom?

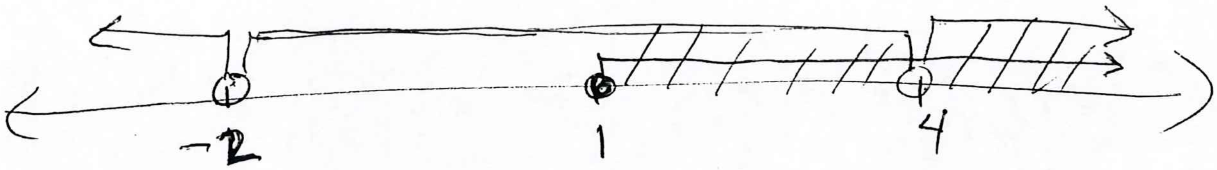
$(x-1)^2-9 \neq 0$, or $(x-1)^2 \neq 9$, so $x-1 \neq \pm 3$.

and $x \neq 1 \pm 3$ or $x \neq 4$ or -2.

$D_{f(g)}$: $\boxed{(-\infty,-2)\cup(-2,4)\cup(4,\infty)}$

Continued →

Graph these two domains + determine their intersection. This is the dom $f \circ g$:



$D_{f \circ g}$: $[1, 4) \cup (4, \infty)$