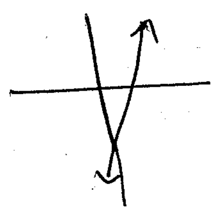
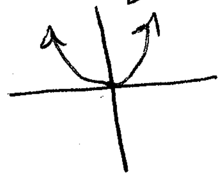
Inverse of a function $f(x)$

For a fcn. $f(x)$ to be invertible (to have an inverse) on its domain, it must be ~~one~~ one-to-one.

So, for ex, $f(x) = 3x - 5$ is invertible on $\mathbb{R}$ but $f(x) = x^2$ is not invertible on $\mathbb{R}$.
The line $f(x) = 3x-5$ is one-to-one, but the parabola $f(x) = x^2$ is not.

We would see this quickly by graphing the two

But suppose you don't know these off the bat.
You could apply the definition of 1-1 to $y = x^2$ to find that ~~if~~ it is not 1-1, since, for $x_1, x_2 \in \operatorname{dom} f$, $f(x_1) = f(x_2)$ does <u>not</u> imply $x_1 = x_2$, as it must to be 1-1.

$$x_1^2 = x_2^2 \longrightarrow \pm\sqrt{x_1^2} = \pm\sqrt{x_2^2} \rightarrow x_1 = \pm x_2$$

Hence $x_1 \neq x_2$ since it also equals $-x_2$.

But we can restrict the domain of a non 1-1 fcn like $f(x) = x^2$ so the new fcn. is 1-1 and hence invertible.

Let $f(x) = x^2$ be a fcn. on $[0, \infty)$
Since x is non-negative, it will pass the horizontal line test, and so it is 1-1.

Find f^{-1} of $f(x) = x^2$ on $[0, \infty)$
and then show $f \circ f^{-1} = f^{-1} \circ f = x$, indicating you've found f correctly.