Sec 6 - Limits - cf. "Zeno's Paradox"

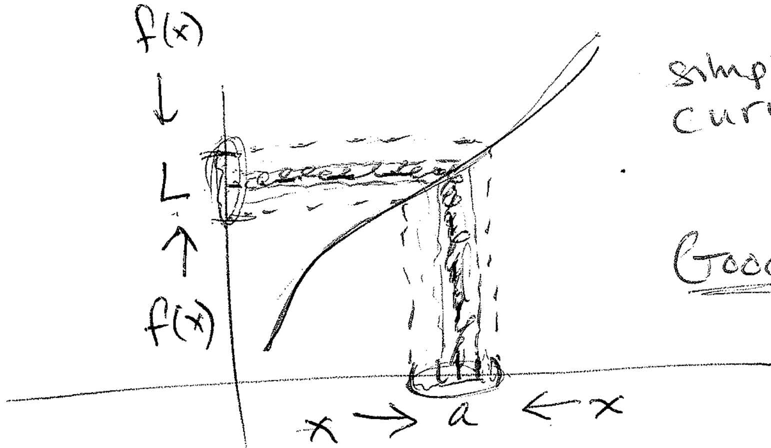
Zeno's Paradox

Def of "limit of f(x) as ~~x~~ x approaches a":
This definition ~~is~~ is elusive until many examples are seen. Here it is:

> The "limit of f(x) as x approaches some value a" is the value L" if, as x gets arbitrarily close (here it means "close as we like" and we mean "ever closer") to a, then f(x) gets arbitrarily close to L.
> ever closer to

"Limit" only makes sense when ~~[illegible]~~ x is very near to a. It's not sensible to consider values of x that are any "large" distance from a. "Near" in analysis is always in terms of some "very small" ε value, $\varepsilon > 0$, or δ value, $\delta > 0$

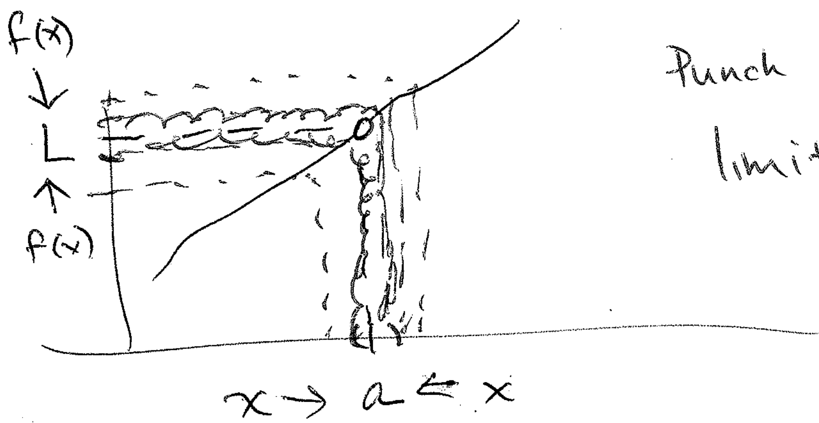
We explore ~~nearness~~ graphically rather than with $\varepsilon, \delta > 0$ algebra.



simple curve

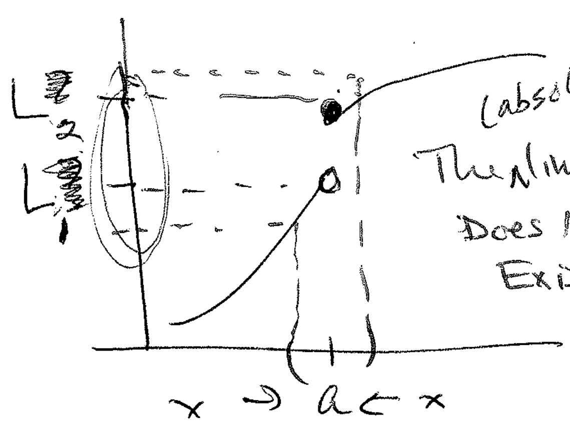
Claim – "L" is the limit as x goes to a

Good. Limit is L.

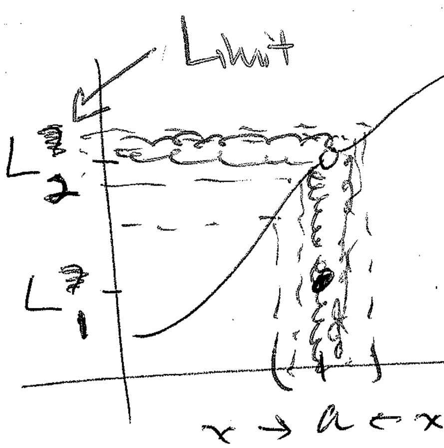


Punch a hole in curve:

limit is still L.



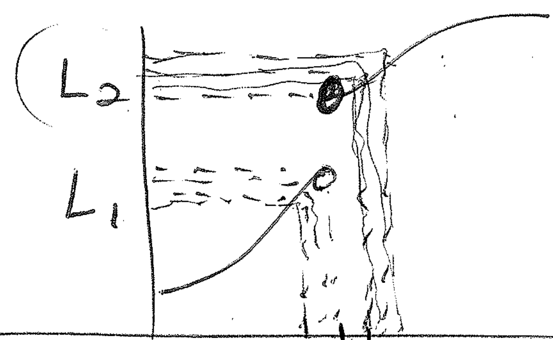
(absolute)
The limit Does Not Exist



The limit does exist. The hole does not affect the limit.

Notation: $$\boxed{\lim_{x \to a} f(x) = L}$$

We can modify our requirement on when a limit exists. Let's allow for a "one-sided limit."



$\lim_{x \to a^-} f(x) = LHL$

$\lim_{x \to a^+} f(x) = RHL$

from the left $x \to a \leftarrow x$ from the right

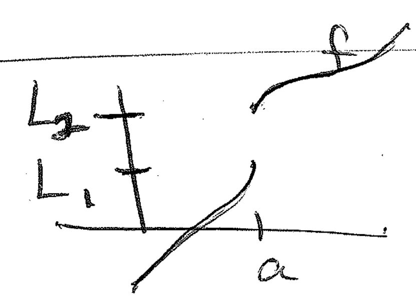
Terminology

$x \to a^-$ x approaches from the left (on the left)

$x \to a^+$ x approaches from the right (on the right

$LHL \equiv$ left-hand limit

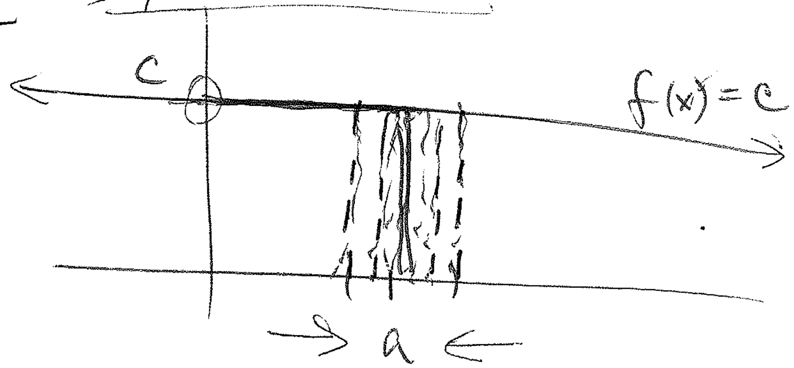
$RHL \equiv$ right-hand limit



$\lim_{x \to a^+} f(x) = L_2$

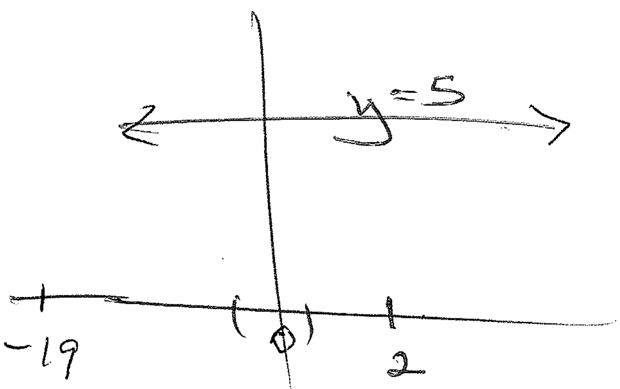
$\lim_{x \to a^-} f(x) = L_1$

Thm - Properties of limits



$$\boxed{\lim_{x \to a} c = c}$$

Ex $f(x) = 5$



$$\lim_{x \to 2} 5 = 5$$

$$\lim_{x \to 0} 5 = 5$$

$$\lim_{x \to -19} 5 = 5$$

Wait a sec:

To say "$\lim_{x \to a} f(x) = L$" is to say

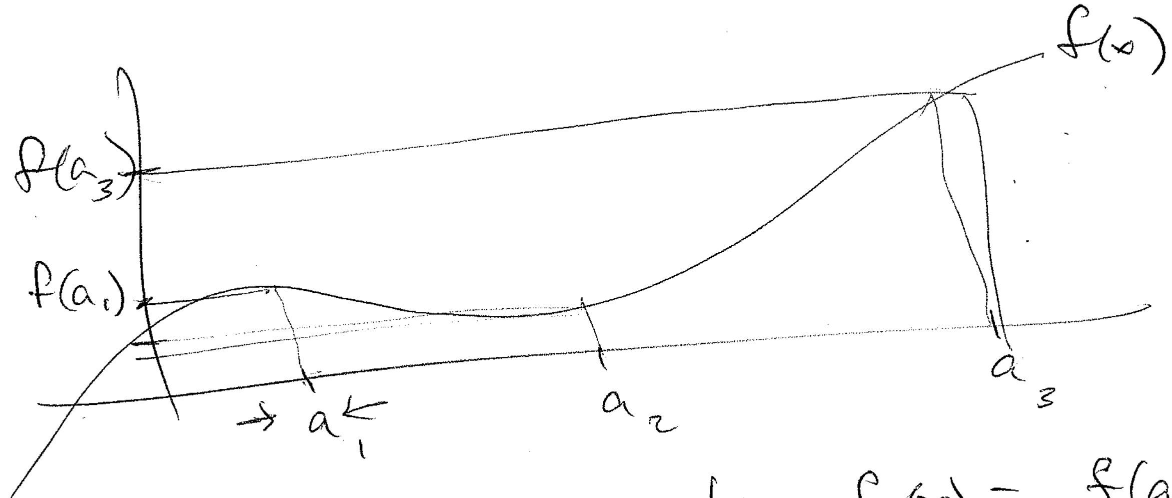
that $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$

In other words, when the LHL = RHL, then the "absolute limit" of $f(x)$ as $x \to a$ "in the absolute sense" exists. "Absolutely" means ~~[illegible]~~ from either side.

~~sides~~

~~Printed at FedEx Office~~

Favorite fcn. - polynomials - we like a fcn w/o breaks, sharp corners, asymptotes. Polynomials do the job.



$$\lim_{x \to a_1} f(x) = f(a_1), \quad \lim_{x \to a_2} f(x) = f(a_2)$$

$$\lim_{x \to a_3} f(x) = f(a_3).$$

So, for polynomials, fcns have limits, as $x \to a$, that correspond to that value of a; ie, $f(a)$

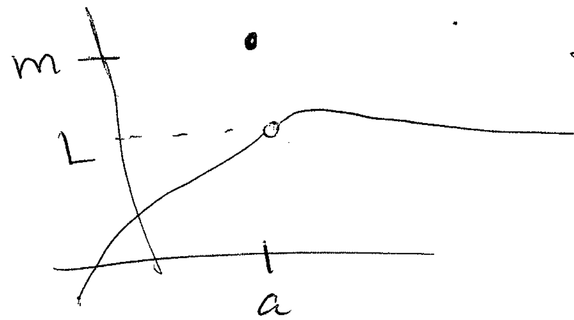
$$\lim_{x \to a} f(x) = f(a)$$

This feature of polynomials is indicative of their continuity. These fcns are said to be "continuous at every $a \in$ domain"

(See 7 - wait a bit)

How does a limit look for a fcn with the following traits:

<u>Hole</u>

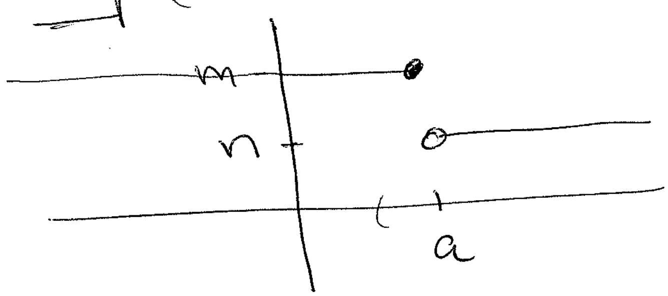


$f(a) = m$

<u>but</u> $\lim_{x \to a} f(x) = L$

<u>Note</u>: $f(a) \neq L$, so, $f(x)$ is not continuous at $x = a$.

<u>Gap</u> (step)



$f(a) = m$

$\lim_{x \to a} f(x)$ does not exist (DNE)

because $LHL \neq RHL$

~~Asymptote~~

LHL: $\lim_{x \to a^-} f(x) = m$ $\quad m \neq n$

RHL: $\lim_{x \to a^+} f(x) = n$

So $\lim_{x \to a} f(x)$ DNE (i.e., in the absolute sense)

Continue by reading them with all the properties like $\lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = \lim_{x \to a} (f \pm g)(x)$

and $\lim_{x \to a} c f(x) = c \lim_{x \to a} f(x)$