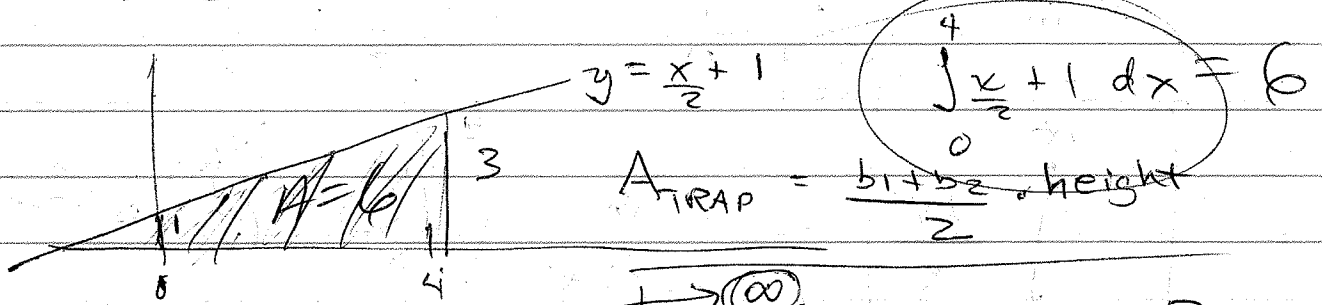


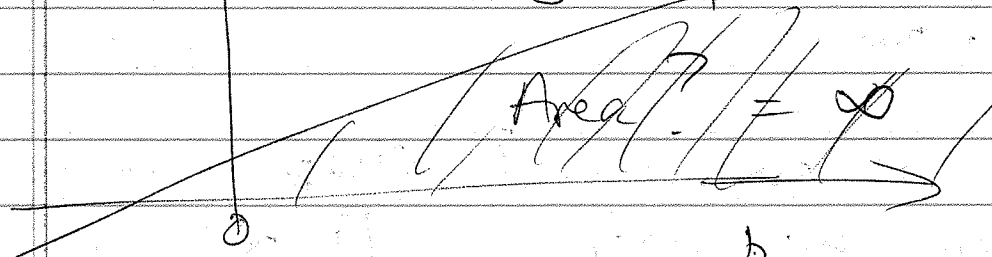
1

Sec 36 - Improper Integrals



$\int_0^4 \frac{x}{2} + 1 dx = 6$

Improper integral $\int_0^{\infty} \frac{x}{2} + 1 dx = ?$



Rewrite as $\lim_{b \rightarrow \infty} \int_0^b \frac{x}{2} + 1 dx$

$\int_0^b \frac{x}{2} + 1 dx = \frac{x^2}{4} + x \Big|_0^b = \frac{b^2}{4} + b - 0$

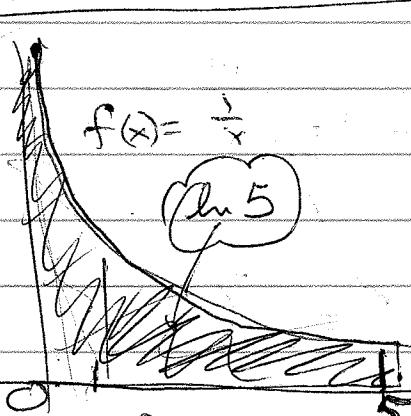
Take limit

$\lim_{b \rightarrow \infty} \left(\frac{b^2}{4} + b \right) = \infty + \infty = \infty$

(clearly)

Area under a line on $(0, \infty) = \infty$

Classic



$\int_1^5 \frac{1}{x} dx = \ln|x| \Big|_1^5 = \ln 5$

$\int_0^5 \frac{1}{x} dx = ?$

$\lim_{a \rightarrow 0^-} \ln|5| - \ln|a| = \ln 5 - (-\infty) = \infty$

②

More likely question (for the sake of series)

Sum $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots = ?$

$\int_1^{\infty} \frac{dx}{x} = \lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x} = \lim_{b \rightarrow \infty} (\ln b - \ln 1)$

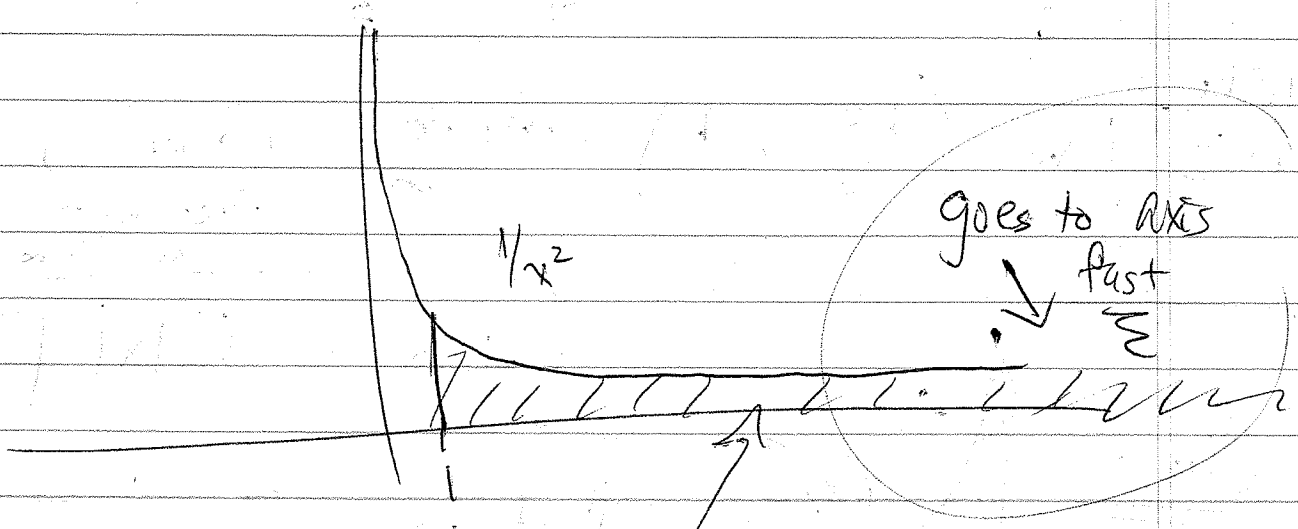
$= \lim_{b \rightarrow \infty} (\ln b - \ln 1) = \ln \infty - 0 = \infty$

Def

We say that an integral ~~where~~ that is infinite on an interval $[1, \infty)$ is divergent.

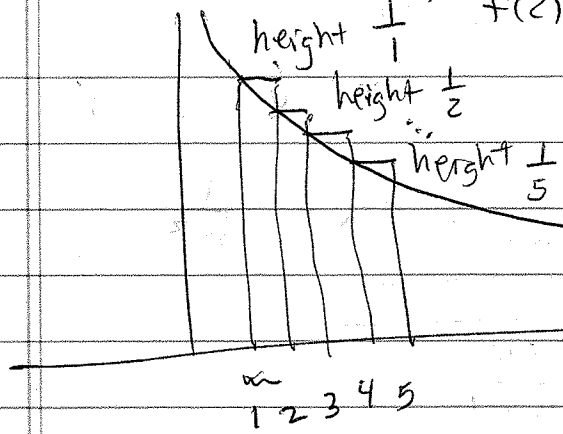
Go to ③

Consider now $f(x) = \frac{1}{x^2}$



Area = ?

$f(x) = \frac{1}{x}$, $f(1) = 1$
 $f(2) = \frac{1}{2}$



$\sum_{x=1}^{\infty} \text{areas height} \times 1$
 $= \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots$

This sum is infinite.

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \dots$
 $1 + \frac{26}{24} > 1$

$= 1 + 1 + \dots = \infty$

42
35
30
107

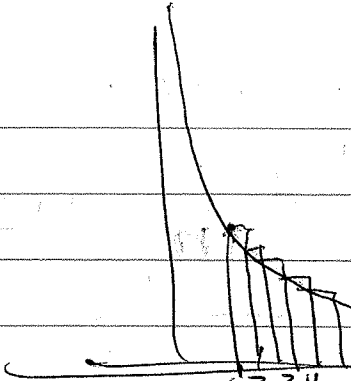
$\sum_{n=1}^{\infty} \frac{1}{n} = \infty \iff \int_1^{\infty} \frac{dx}{x} = \infty$

$f(x) = \frac{1}{x^2}$ → The denominator is growing a lot faster than it does for $\frac{1}{x}$.

Look at the sum of the rect. areas first:

$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$

Can we similarly group these fractions to give $1 + 1 + 1 + \dots$ until the sum appears to be infinite? NO!



$$f(1) = \frac{1}{1^2} = 1, A_1 = 1 \times 1 = 1$$

$$f(2) = \frac{1}{2^2} = \frac{1}{4}, A_2 = \frac{1}{4} \times 1 = \frac{1}{4}$$

$$\vdots$$

$$f(n) = \frac{1}{n^2}, A_n = \frac{1}{n^2}$$

Area = $\underbrace{\text{height}}_{f(x)} \times \underbrace{\text{base}}_{\Delta x = 1}$

$$\text{Area} = \int_1^{\infty} \frac{dx}{x^2} = \lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^2} = ?$$

~~$$\lim_{b \rightarrow \infty} \left(\frac{-1}{x} \right) \Big|_1^b = \lim_{b \rightarrow \infty} \left(-\frac{1}{x} - \frac{-1}{b} \right)$$~~

~~$$= \lim_{b \rightarrow \infty} \left. \frac{-1}{x} \right|_1^b = \lim_{b \rightarrow \infty} \left(\frac{-1}{b} - \frac{-1}{1} \right)$$~~

$$= \lim_{b \rightarrow \infty} \left(\frac{-1}{b} - \frac{-1}{1} \right) = \lim_{b \rightarrow \infty} \left(1 - \frac{1}{b} \right) \neq 1$$

$$= \lim_{b \rightarrow \infty} 1 - \lim_{b \rightarrow \infty} \frac{1}{b} = 1 - \frac{1}{\infty} \rightarrow 1$$

= 1

→ What does this represent?
Go to (2) for answer.

(5)

Def An integral that is finite on an interval $[-\infty, \infty]$ is said to converge.

When we have a two-sided limit, i.e.

$$\int_{-\infty}^{\infty} f(x) dx, \text{ we look at}$$

each half, $\int_{-\infty}^0 f(x) dx + \int_0^{\infty} f(x) dx$

If ~~both~~ integrals converge then so does $\int_{-\infty}^{\infty} f(x) dx$.

But if only one converges + the other diverges, then $\int_{-\infty}^{\infty} f(x) dx$ diverges.

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