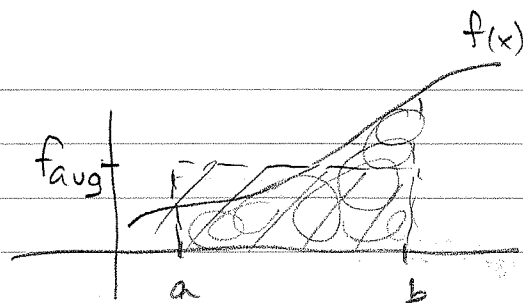


Sec 35 HW

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$$



The avg. value of the function on $[a, b]$ is the height of the rectangle whose area is equal to the area under the curve (the def. integral) on $[a, b]$. See the sketch.

#1d) f_{avg} on $[1, 2]$ of $f(x) = \sqrt{x+1}$

$$f_{\text{avg}} = \frac{1}{2-1} \int_1^2 \sqrt{x+1} dx = \frac{2(x+1)^{3/2}}{3} \Big|_1^2 = \frac{2}{3} (3^{3/2} - 2^{3/2})$$

$$= \frac{2}{3} (\sqrt{27} - \sqrt{8}) = \frac{2}{3} (3\sqrt{3} - 2\sqrt{2})$$

$$= \boxed{\frac{2\sqrt{3} - 4\sqrt{2}}{3}}$$

#2) $P(t) = -4t^3 + 20t + 400$, P_{avg} on $[0, 5] = ?$

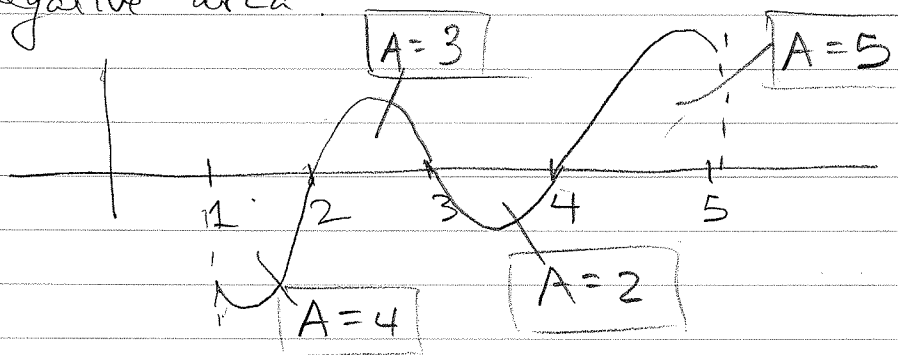
$$P_{\text{avg}} = \frac{1}{5-0} \int_0^5 (-4t^3 + 20t + 400) dt$$

$$= \frac{1}{5} (-t^4 + 10t^2 + 400t) \Big|_0^5 = \frac{1}{5} (-625 + 250 + 2000)$$

$$= \frac{1}{5} (1625) = \boxed{\$325}$$

Note: $P(0) = \$400$. That is, before it hits the market & shortly thereafter, the price is \$400.

#4a) Refers to the graphs in Sec 34. Since f_{avg} on $[a, b] = \frac{\int_a^b f dx}{b-a}$ we simply take the areas given and divide by $b-a$. However, since f_{avg} is a fun. value, the sign is important. So if fun lies below x line, we use the negative area.



$$f_{avg} = \frac{-4 + 3 - 2 + 5}{5-1} = \frac{2}{4} = \frac{1}{2}$$

#5a) This problem is a straight application of the definite integral. Blueberries are thawing from -5°C (given freezer temp) to 20°C (assume they will reach room temp). (The temps are not the variable)

Given is the rate of temp increase: $\frac{dT}{dt} = 10e^{-.4t}$

Setting up the integral over $[0, \frac{1}{4}]$ time in hours:

$$\frac{dT}{dt} = 10e^{-.4t} \rightarrow dT = 10e^{-.4t} dt \rightarrow T = \int_0^{\frac{1}{4}} 10e^{-.4t} dt$$

$$= \frac{10}{-.4} \left(e^{-.4(\frac{1}{4})} - e^{-.4(0)} \right) = \left(-25e^{-.1} + 25 \right)$$

temp change

Remember, $T = \int_0^{1/4} 10e^{-.4t} dt$ is total change in temp in $1/4$ hr. (the integral of the rate of temp change is the temp. fun $T(t)$, but total change on $[0, 1/4]$ is the area under the fun). Since the temp at the start was -5°C , the temp reached after $1/4$ hr is

$$= -5 + \int_0^{1/4} 10e^{-.4t} dt = -5 + [-25e^{-.4t}]_0^{1/4} = -5 + (-25e^{-.1} + 25) = -25e^{-.1} + 20^\circ\text{C}$$

$$\approx -2.62^\circ\text{C}$$

Notice the room temp is not needed to solve.

b) Added question: What is T_{avg} on $[0, 1/4]$?

To answer this we need the temp fun $T(t)$. We only have the temp change $\int 10e^{-.4t} dt$.

How can we find the fun $T(t)$?

For this we need to solve the indefinite integral for the constant based on the initial condition $T(0) = -5^\circ\text{C}$

$$\int T(t) = \int 10e^{-.4t} dt = -25e^{-.4t} + C$$

$$T(0) = -5 = -25e^{-.4(0)} + C$$

$$-5 = -25 \cdot 1 + C$$

$$C = 20$$

$$\boxed{T(t) = -25e^{-.4t} + 20}$$

★ Notice that $T(1/4) = -2.62^\circ\text{C}$, which we got by adding the value of the definite integral to -5 .

We didn't need the T fn. to solve for part a, but we do for finding T_{avg} .

$$\overline{T}_{\text{avg}} = \frac{1}{1/4} \int_0^{1/4} (-25e^{-.4t} + 20) dt$$

$$= 4 \left(\frac{-25}{-.4} e^{-.4t} + 20t \right) \Big|_0^{1/4}$$

$$= 4 (62.5 e^{-1} + 5 - 62.5 e^0 - 20(0)) \approx \boxed{-3.8^\circ\text{C}}$$

#6 Production rate $\frac{dH}{dt} = 30\sqrt{t}$

a) Given $H'(t)$, find the number of hoes produced in first 36 weeks.

Like #5, this is a rate we integrate to get total change, then add to zero (no backhoes at $t=0$).

$$\frac{dH}{dt} = 30\sqrt{t} \Rightarrow \int dH = \int 30\sqrt{t} dt \rightarrow H = \int 30\sqrt{t} dt$$

Since we'll need $H(t)$ to do part (b), the average, we will find the actual fn. First from the definite integral and the initial condition $H(0) = 0$.

$$H(t) = \int 30\sqrt{t} dt = \frac{30}{3/2} t^{3/2} + C = \boxed{20t^{3/2} + C}$$

$$H(0) = 20(0^{3/2}) + C = 0; C = 0$$

$$H(t) = 20t^{3/2}; \quad H(36) = \boxed{4320 \text{ hoes}}$$

$$\#6b) \quad H_{\text{avg}} \text{ on } [0, 36] = \frac{1}{36-0} \int_0^{36} 20t^{3/2} dt$$

$$= \frac{1}{36} (20 \cdot 36^{3/2} - 20 \cdot 0^{3/2}) = \boxed{120 \text{ backfires}}$$

If you did part (a) like $\int_0^{36} 30\sqrt{t} dt$, you got the answer, but that wouldn't give you the fn. to integrate for H_{avg} . This is an important point!

$$\#7. \quad C'(x) = 12x + 20, \quad \int_5^{10} C'(x) dx = \int_5^{10} 12x + 20 dx$$

(x = million paper clips)

$$= \left. \frac{12x^2}{2} + 20x \right|_5^{10} = 600 + 200 - 150 - 100$$

$$= \boxed{\$ 550}$$

#8a) The next problem is like #5 and #6, in that we are given a rate fn. (the rate at which the value of an investment fund changes over time, based on its performance since inception). We are asked for the change in the value from year 1 to 8.

$$\frac{dV}{dt} = 3\sqrt{t} \rightarrow \int dV = \int 3\sqrt{t} dt \rightarrow V = \int 3\sqrt{t} dt$$

indefinite integral gives the value fn. itself;

$$V(t) = \frac{3t^{3/2}}{3/2} + C = 2t^{3/2} + C$$

The initial condition given is $V(1) = 20$

$$\text{So } V(1) = 2(1^{3/2}) + C = 20, \quad C = 18$$

Thus, $V(t) = 2t^{3/2} + 18$ is the stock's value fcn.

(Note: $V(1) = 2 \cdot 1 + 18 = 20$)

Basil buys ^{at year 1} a share at \$20. Its value 8 years later (year 9) is given by

$$\$20 + \int_1^9 3\sqrt{t} \, dt = \$20 + \left. \frac{3t^{3/2}}{3/2} \right|_1^9 = \$20 + 2t^{3/2} \Big|_1^9$$

$$= \$20 + 2(9^{3/2}) - 2(1^{3/2}) = \boxed{\$72}$$

b) The average value of a share in this time is given by the usual formula:

$$V_{\text{avg}} \text{ on } [1, 9] = \frac{1}{9-1} \int_1^9 (2t^{3/2} + 18) \, dt$$

$$V_{\text{avg}} = \frac{1}{8} \left(\frac{2t^{5/2}}{5/2} + 18t \right) \Big|_1^9$$

$$= \frac{1}{8} \left(\frac{4}{5} (9^{5/2}) + 18(9) - \frac{4}{5} (1^{5/2}) + 18(1) \right)$$

$$= \frac{1}{8} (194.4 + 162 - 15.2) = \frac{341.2}{8} = \boxed{\$42.70}$$

★ Notice that we did 2 integrations:
The first to get the fcn. $V(t)$ using dV/dt & initial conditions; the second to get average value using $V(t)$.

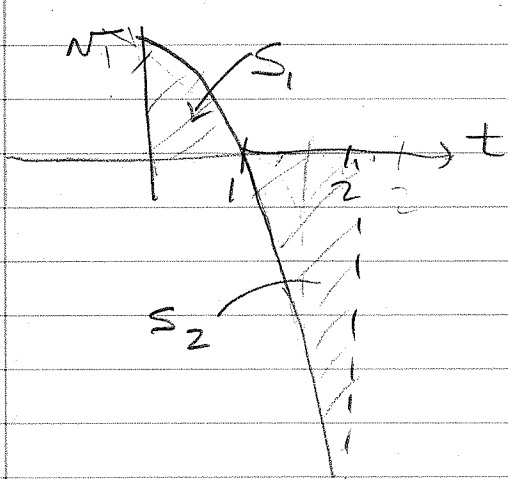
#9 $v(t) = 1 - t^2$ particle velocity

a) $\left| \int_{t_1}^{t_2} v(t) dt \right| =$ displacement from t_1 to t_2

$$\left| \int_0^2 (1 - t^2) dt \right| = \left| t - \frac{t^3}{3} \Big|_0^2 \right| = \frac{2}{3} \text{ mile}$$

b) $\int_{t_1}^{t_2} v(t) dt =$ total distance
(area under curve from t_1 to t_2)

The fun. is a parabola, with roots at ± 1 . We only need $t = 1$ hr to determine where the fun. crosses the axis.



$$\int_0^1 (1 - t^2) dt + \int_1^2 (1 - t^2) dt$$

Reverse \rightarrow
the limits of \int
since the def int
will have a negative
value on $[1, 2]$

$$\begin{aligned} \text{Total dist} &= S_1 + S_2 = t - \frac{t^3}{3} \Big|_0^1 + t - \frac{t^3}{3} \Big|_1^2 \\ &= 1 - \frac{1}{3} + 1 - \frac{1}{3} - 2 + \frac{8}{3} = \frac{6}{3} = 2 \end{aligned}$$

$\frac{1}{2} \quad \frac{1}{2}$ 2 miles

#11 Set up only

1) present value
2) accumulated (future) value

$$PV = \int_0^T f(t) e^{-rt} dt$$

$$FV = e^{rT} \int_0^T f(t) e^{-rt} dt$$

a) where $T = 10$, $r = .06$, $f(t) = 40,000 + 200t$

$$PV = \int_0^{10} (40,000 + 200t) e^{-.06t} dt$$

$$FV = e^{.06(10)} \int_0^{10} (40,000 + 200t) e^{-.06t} dt$$

These require some IBP

b) where $T = 10$, $r = .06$, $f(t) = 5000e^{.01t}$

$$PV = \int_0^{10} 5000 e^{.01t} e^{-.06t} dt$$

$$FV = e^{.06(10)} \int_0^{10} 5000 e^{.01t} e^{-.06t} dt$$

#7. The trick is to simplify correctly.

Also, memorize that $\int_0^T e^{at} dt = \frac{e^{at}}{a} \Big|_0^T$

#12 $f(t) = \$4000$, $r = .07$, $T = 6$

Value of acct after 6 years (i.e. FV)

$$\begin{aligned}
 FV &= e^{.07(6)} \int_0^6 4000 e^{-.07t} dt \\
 &= 4000 e^{.42} \int_0^6 e^{-.07t} dt \\
 &= \frac{4000 e^{.42}}{-.07} \left(e^{-.07t} \Big|_0^6 \right) \\
 &= \frac{4000}{-.07} \left(e^{-.42} - e^0 \right) = \frac{4000}{.07} \left(1 - \frac{1}{e^{.42}} \right)
 \end{aligned}$$

For the test this would be enough.

#15. Since money will flow at a constant rate of $\$4000/\text{yr.}$ for 6 years at 6% compound interest, we can find the interest over 6 months ($T = \frac{1}{2}$)

by taking FV minus the aunt's contribution over those 6 months:

$$\begin{aligned}
 \text{Interest} &= FV - \frac{\$4000}{2} \\
 &= e^{.06(\frac{1}{2})} \int_0^{\frac{1}{2}} 4000 e^{-.06t} dt - \$2000
 \end{aligned}$$

↑
aunt's contribution over 1/2 year

$$= \frac{\$4000 \cdot e^{.03}}{-.06} \left[e^{-.06t} \right]_0^{1/2} - \$2000$$

$$= \frac{\$4000 e^{.03}}{-.06} (e^{-.03} - e^0) - \$2000$$

$$= \frac{\$4000}{.06} (1 - e^{-.03}) - \$2000$$

$$= \$30.30$$

(The book's simplification is psychotic, though correct.)

Mine is reasonable. On a test you only need the second to last line.)
or E