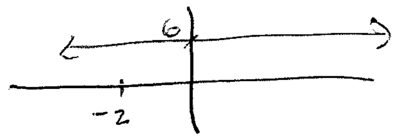
HW 4 – Limits (Sec 6)

1. $\lim_{x \to -2} 6 = \boxed{6}$ b/c $f(x) = 6$ is a const fcn



2. $\lim_{x \to 2} (3x^2 + 5x + 2) = \boxed{24}$ b/c polynomial limits are found by just evaluating the fcn. at the value x goes to

3. $\lim_{s \to 0} (2s^3 - 1)(2s^2 + 4) = \boxed{-4}$ also a polynomial

4. $\lim_{x \to 0} \frac{2x-3}{2x-1} = \frac{-3}{-1} = \boxed{3}$ the fcn is defined at $x=0$ so there's no issue

5. $\lim_{x \to 3} \frac{x^2-16}{x-4} = \lim_{x \to 3} x+4 = \boxed{7}$ same as #4, though $x \neq 4$, the value x goes to is 3

6. $\lim_{x \to 2} \frac{x^2+x-6}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}} = \lim_{x \to 2} x+3 = \boxed{5}$

7. $\lim_{x \to 2} \frac{\sqrt{x}-2}{x-4} = \frac{\sqrt{2}-2}{2-4} = \frac{\sqrt{2}-2}{-2} = \boxed{\frac{2-\sqrt{2}}{2}}$

which is the same as

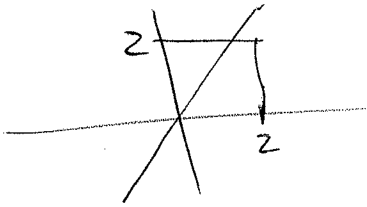
$$\lim_{x \to 2} \left(\frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} \right) = \lim_{x \to 2} \frac{(\cancel{x-4})}{(\cancel{x-4})(\sqrt{x}+2)} = \lim_{x \to 2} \frac{1}{\sqrt{x}+2}$$

(method of conjugation)

$$= \boxed{\frac{1}{\sqrt{2}+2}}$$

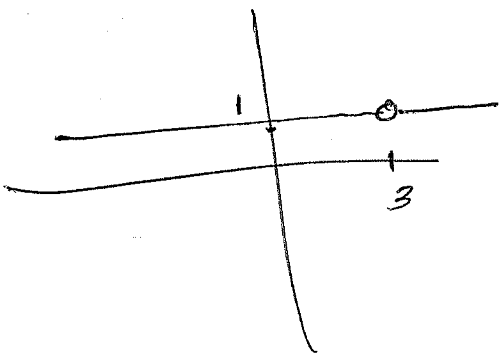
8. $\lim\limits_{x \to 1} \dfrac{[-1/(x+3)] + 1/4}{x}$ Straight computation gives $\dfrac{-1/4 + 1/4}{1} = \boxed{0}$

9. $\lim\limits_{x \to 2} x = \boxed{2}$



10. $\lim\limits_{x \to 3} e^{2x-1} = \boxed{e^5}$

11. $\lim\limits_{x \to 3} \dfrac{x-3}{x-3}$



Since evaluating at $x = 3$ gives $0/0$, which is an "indeterminate" form, you want to try algebra. Obviously $(x-3)/(x-3) = 1$, but Dom $f(x)$ is $x \neq 3$. Nevertheless, as $x \to 3^+$ and as $x \to 3^-$, $\lim f(x) = \boxed{1}$

12. $\lim\limits_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$ The dom excludes $h = 0$, but look at the algebraically simplified function to see the actual graph.

$\lim\limits_{h \to 0} \dfrac{(x^2 + 2xh + h^2 - x^2)}{h}$

$=$

$\lim\limits_{h \to 0} \left(\dfrac{2xh + h^2}{h} \right) = \lim\limits_{h \to 0} \dfrac{2x + h}{1} = 2x$

So, limit of the function is the expression $\boxed{2x}$

In fact, this expression is the derivative of $f(x) = x^2$.

We say that if this limit exists, the fcn $f(x)$ is "differentiable"

13. $\lim_{h\to 0} \frac{\sqrt{h+9}-3}{h}$

Again, $h \neq 0$, so to get a form we can work with to see what is going on <u>near</u> $h=0$, use the algebraic technique of multiplying top + bottom by the conjugate of the top

$$\frac{(\sqrt{h+9}-3)(\sqrt{h+9}+3)}{h(\sqrt{h+9}+3)}$$

$$= \frac{h+9-9}{h(\sqrt{h+9}+3)} = \frac{h}{h(\sqrt{h+9}+3)} = \frac{1}{\sqrt{h+9}+3}$$

whose limit as $h \to 0$ is $\boxed{1/6}$

14. $\lim_{y\to 0} \frac{6y-9}{y^3-12y+3}$

Straight computation works since there's no issue at $y=0$

$$\lim_{y\to 0} f(y) = \frac{-9}{3} = \boxed{-3}$$

15. $\lim_{x\to 2} \frac{2-x}{\sqrt{7+6x^2}} = \frac{0}{\sqrt{7}} = 0$

16. $f(x) = \begin{cases} \frac{x^2-1}{x+1}, & x < -1 \\ x^2-3, & x \geq -1 \end{cases}$

<u>Aside</u> - <u>important</u>

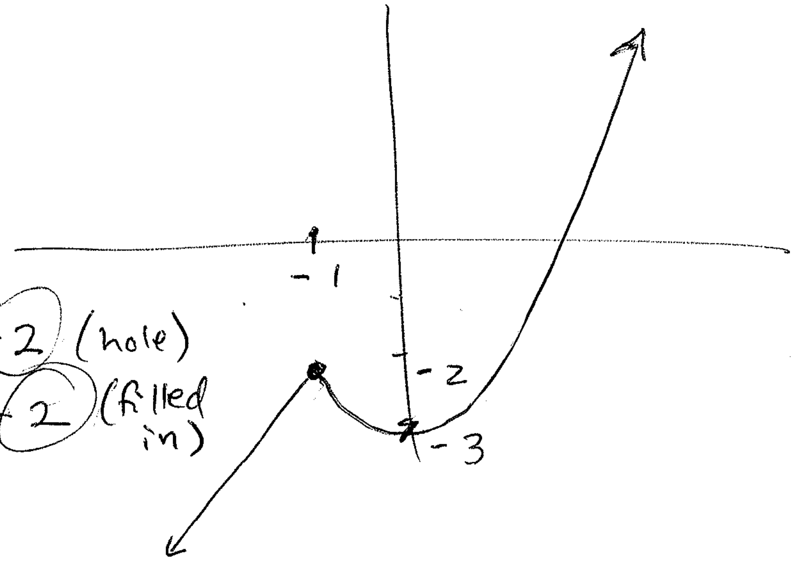
We always want to see if the graph is connected or at least has at worst a hole at the domain's interval endpts. To do this, evaluate each piece at the "break"

Simplify:

$$f(x) = \begin{cases} x-1, & x < -1 \\ x^2-3, & x \geq -1 \end{cases}$$

The break is at $x=-1$, which is where the limit is sought.

$$\lim_{x \to -1} \begin{cases} x-1, & x<-1 \\ x^2-3, & x \geq -1 \end{cases}$$



Piece one: at $x=-1$ is -2 (hole)

Piece two: at $x=-1$ is -2 (filled in)

Apparently, the pieces meet up. But even if they didn't, $\lim_{x \to -1} f(x) = \boxed{-2}$

17. $f(x) = \begin{cases} 3+x, & x<2 \\ 3x+1, & x \geq 2 \end{cases}$ find $\lim_{x \to 2} f(x)$

The first piece of $f(x)$ is where we look at LHL

$\lim_{x \to 2^-} (3+x) = \boxed{5}$

The second piece of $f(x)$ is where we inspect RHL

$\lim_{x \to 2^+} (3x+1) = \boxed{7}$

RLH $\neq$ LHL

so $\lim_{x \to 2} f(x)$ DNE

22. $\lim\limits_{x \to 1^-} \dfrac{1+x^2}{1-x^2}$ the numerator $\lim\limits_{x \to 1^-} (1+x^2) = 2$

the denominator $\lim\limits_{x \to 1^-} (1-x^2) = 0$

This is like Ex. 6.13.

When the numerator is fixed + the denominator is zero, the value of the fcn. becomes unbounded as $x \to 1$ from either direction. No algebraic technique simplifies it so we can avoid the constant/zero.

But, since x^2 is always (+) and it's less than 1 when $x \to 1^-$, then $1-x^2$ is (+) so $\Rightarrow$ the $\dfrac{\text{numerator}}{\text{denom}} = \dfrac{+}{+} = +\infty$ as $x \to 1^-$

We won't graph this.

23. $\lim\limits_{x \to 0^-} \left(\dfrac{1}{x} + \dfrac{1}{x^2 - x} \right)$ Before concluding it's $\frac{1}{0} - \frac{1}{0}$ which is indeterminate, simplify it with the LCD

$$\lim_{x \to 0^-} \left(\frac{1}{x} + \frac{1}{x(x-1)} \right) = \lim_{x \to 0^-} \left(\frac{x - \cancel{1} + \cancel{1}}{x(x-1)} \right)$$

$$= \lim_{x \to 0^-} \left(\frac{1}{x-1} \right) = \boxed{-1}$$

18. $\lim\limits_{x \to 2} f(x)$ when $f(x) = \begin{cases} 3+x, & x < 2 \\ 3x-1, & x > 2 \end{cases}$

Notice the graph has no pt at $x=2$. i.e., The fcn. is not defined at $x=2$

Straight evaluation of each piece gives

$\lim\limits_{x \to 2^-} (3+x) = 5 = LHL$

and $\lim\limits_{x \to 2^+} (3x-1) = 5 = RHL$

So $LHL = RHL = \boxed{5}$

19. $\lim\limits_{x \to 4} \dfrac{x^2-16}{x-4} = \lim\limits_{x \to 4} x+4 = \boxed{8}$

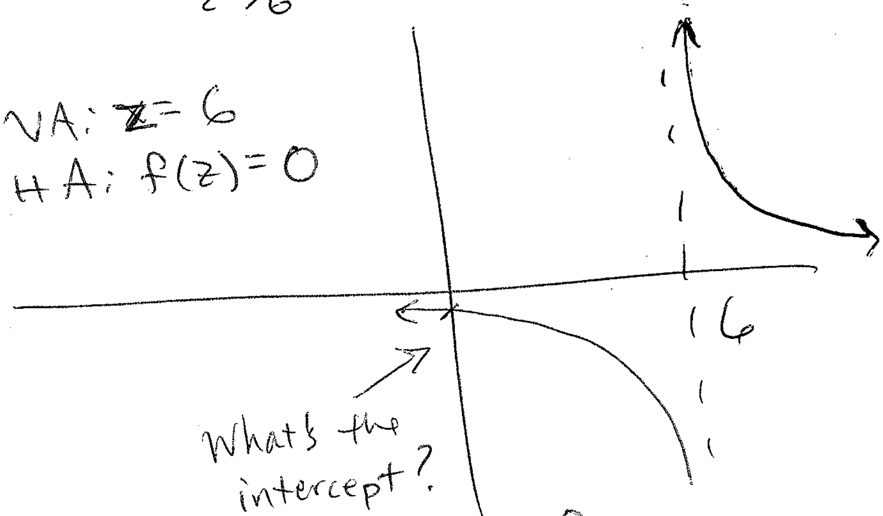
Can you graph it? $f(x) = \dfrac{x^2-16}{x-4}$

20. $\lim\limits_{z \to 6} \dfrac{z-6}{z^2-36} = \lim\limits_{z \to 6} \dfrac{1}{z+6} = \boxed{\dfrac{1}{12}}$

21. $\lim\limits_{z \to 6} \dfrac{z+6}{z^2-36} = \lim\limits_{z \to 6} \dfrac{1}{z-6}$

Since we get $\frac{1}{0}$, we need to inspect near $z=6$ on both sides ... clearly prefer to use a graph than plugging in values on either side of $z=6$

VA: $z=6$
HA: $f(z)=0$



What's the intercept?
Evaluate at $z=0$

24. $\lim_{t \to 9} \dfrac{t-9}{\sqrt{t}-3} = \lim_{t \to 9} \left(\dfrac{t-9}{\sqrt{t}-3} \cdot \dfrac{\sqrt{t}+3}{\sqrt{t}+3} \right)$ Done ⦚

$= \lim_{t \to 9} \dfrac{t\sqrt{t} + 3t - 9\sqrt{t} - 27}{t-9} = \dfrac{27+27-27-27}{9-9}$

$\sqrt{t}^2 - 3^2$ $= 0/0$ indeterminate still

But, if you are either masochistic or not thinking:

This one is tricky. Remember on the first quiz we factored $x^2 - 11$ as $(x+\sqrt{11})(x-\sqrt{11})$

This is a similar situation. We can "factor"

$t-9$ as $(\sqrt{t}+3)(\sqrt{t}-3)$.

Then $\lim_{t \to 9} \dfrac{(\sqrt{t}+3)(\sqrt{t}-3)}{1(\sqrt{t}-3)} = \boxed{6}$

(The graph has a hole at $t=9$)

Update on # 24 – If I didn't expand the numerator at first, I'd get the cancellation!

25. $\lim_{h \to 0} \dfrac{\sqrt{h^2+4} - 2}{4} = \dfrac{2-2}{4} = \boxed{0}$

26. $\lim_{x \to 4} \dfrac{x^2-16}{\sqrt{x+5}-3} = \dfrac{0}{0}$ indeterminate – see Ex 6.9 (conjugation again)

$\lim_{x \to 4} \left(\dfrac{x^2-16}{\sqrt{x+5}-3} \cdot \dfrac{\sqrt{x+5}+3}{\sqrt{x+5}+3} \right) = \lim_{x \to 4} \dfrac{\ldots}{x+5-9}$

"a − b" "a+b"

$$= \lim_{x\to 4} \frac{\overbrace{(x+4)\cancel{(x-4)}}^{x^2-16}\left(\sqrt{x+5}+3\right)}{\cancel{(x-4)}} = \lim_{x\to 4} \frac{(x+4)\left(\sqrt{x+5}+3\right)}{1}$$

$$= (8)(6) = \cancel{\boxed{14}}\ \boxed{48}$$

27. $\lim\limits_{x\to 5} \left(\frac{x^2+x-30}{2x-10}\right) = \lim\limits_{x\to 5} \frac{\cancel{(x-5)}(x+6)}{2\cancel{(x-5)}} = \boxed{\frac{11}{2}}$

28. $\lim\limits_{x\to 3} \left(\frac{\frac{1}{x}-\frac{1}{3}}{x^2-9}\right) = \lim\limits_{x\to 3}\left[\frac{\left(\frac{3-x}{3x}\right)}{(x+3)(x-3)}\right] = \lim\limits_{x\to 3}\left[\frac{-1\,*}{3x(x+3)}\right]$

$*$ The $\frac{3-x}{x-3}$ cancels to -1

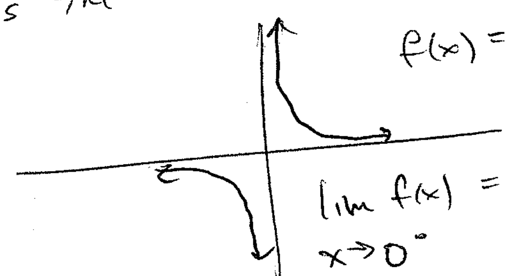
$= \boxed{\frac{-1}{54}}$

29. $\lim\limits_{x\to -1} \frac{x^2+3x+2}{x^2+x-6} = \frac{0}{-6} = \boxed{0}$

30. $\lim\limits_{x\to 6^-} \left(\frac{-4x+3}{x-6}\right)$

Why is this asking about the limit as x approaches 6 on the left? The VA of the mother function illustrates the situation.

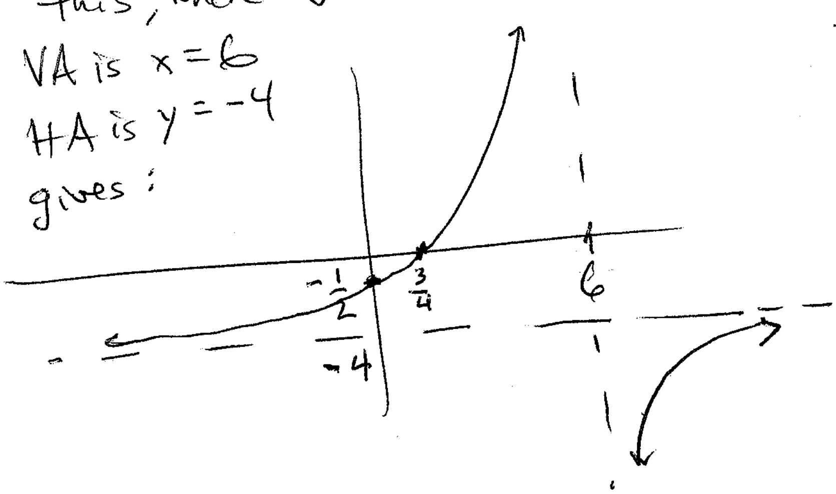
$f(x) = \frac{1}{x}$



$\lim\limits_{x\to 0^-} f(x) = -\infty$

$\lim\limits_{x\to 0^+} f(x) = +\infty$

Graphing this, where VA is $x=6$, HA is $y=-4$ gives:



$\lim\limits_{x\to 6^-} f(x) = \infty$