Math 220-02 Homework - solutions

1. The polynomials are b, d, f, g, i, j
Their degrees are 6, 3, 5, 3, 1, 0

For example, (f) has deg 5 because after expanding it, it has the familiar form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

(f) $(x+1)(x+2)(x^3+4)$

$$= x^{1+1+3} + \cdots + 1 \cdot 2 \cdot 4$$
$$= x^5 + \cdots + 8$$

Notice we don't need to do the entire multiplication to discover the degree. The first terms have product x^5, so the degree is 5.

(i) $\sqrt{2}\,x - \sqrt{3}$ is a polynomial because $\sqrt{2}, \sqrt{3}$ are real. Degree is clearly 1.

(j) This is an example of a constant poly. All $a_i = 0$ except a_0. The degree of any const. poly is 0. from $a_0 x^0 = a_0$

Rational functions are simply the ratio of polynomials, where the bottom poly is not the constant.

So, a, e are the only rational expres.

Finally (e) $\frac{\sqrt{x}}{\sqrt{x+1}}$ is not a ratio of polynomials, so it's neither

(h) $2^x + x^2$ is an exponential plus a polynomial, therefore neither poly nor rational

(k) has non-integer exponents

2. Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$,

find the roots to Ex. 3.3

Doing $f(x) = 2x^2 + 5$ first, notice the discriminant $\sqrt{b^2 - 4ac}$

$$= \sqrt{0^2 - 4(2)(5)} = \sqrt{-10}$$

so it has no (real) roots

For all the others, $b^2 - 4ac > 0$
So they have 2 roots.

ex. $f(x) = x^2 - 2x$,

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(0)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4}}{2} = \frac{2 \pm 2}{2}$$

$$x = \frac{4}{2} \text{ or } \frac{0}{2}, \text{ or } \boxed{2 \text{ or } 0}$$

The factorization, by the way, is

$$x^2 - 2x = x(x-2)$$

Notice this is the same as $(x-0)(x-2)$, which follows the form of factorization of a poly into $(x - r_1)(x - r_2)$.

Here, $r_1 = 0$, $r_2 = 2$

3. Slope $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_1 - y_2}{x_1 - x_2}$

(c) $$\frac{-1/4 - 1/5}{2/3 - 1/7} = \frac{-9/20}{11/21}$$

$$= \frac{-9}{20} \cdot \frac{21}{11} = \frac{-189}{220}$$

4. These problems for finding the eq'n. of the line given certain information relates to the problems about finding a cost function or profit function, given certain info.

The process is the same. You might be looking for the slope or the y-int so you can write the full equation, $y = mx + b$

Ex (c) Through (1, 4) parallel to x-axis.

Since x-axis has slope = 0, so will the line we seek. Using the pt.-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 0(x - 1)$$
$$y - 4 = 0 \longrightarrow \boxed{y = 4}$$

(g) x-intercept is $-2/3$
$\perp$ to $x + y + 1 = 0$

First piece of info says the point $(-2/3, 0)$ is on the line.
Second piece says $m_\perp = -1/m_{given}$

So finding the slope of the given:

$x + y + 1 = 0 \rightarrow y = -x - 1$

$\rightarrow m = -1$ hence, $m_{\perp} = -1/-1 = 1$

$\boxed{m = 1}$ pt. $(-2/3, 0)$

Use pt-slope form:

$y - (0) = 1(x - -2/3)$

$\boxed{y = x + 2/3}$

5. Given that $(2, 9)$ lies on $kx + 3y + 4 = 0$ find k.

Soln Again, it's a matter of using the info to find what's missing.

Using $(x, y) = (2, 9)$, substitute info $0 = kx + 3y + 4$ to solve for k.

$0 = k(2) + 3(9) + 4 \rightarrow 2k = -31$

$\rightarrow \boxed{k = -31/2}$

6. $C(x) = mx +$ fixed costs
where m is the marginal cost of a linear cost fcn.

That is, m = cost of producing each additional item. Later, our cost fcns. won't typically be linear. Then marginal cost will not be slope of a line (const), but will vary with production value x. We'll go into this theory in some detail soon. Remember, x is # of items manufactured.

So, $C(x)$ = cost the manufacturer has for producing x items.

Revenue $R(x) = px$
where p is price/item and x is number of items sold

~~Profit $P(x)$ = Revenue - Cost~~

Finally, Profit = Revenue - Cost

We seek to sell enough so revenue exceeds cost of manufactur

Given $\longrightarrow$ $\boxed{\begin{matrix} m = .40 \\ p = .50 \\ \text{fixed} = \$400 \end{matrix}}$

6a) $C(x) = mx + \text{fixed}$

$C(x) = .40x + 400$

$R(x) = .50x$

$P(x) = R(x) - C(x)$

$= .50x - (.40x + 400)$

$= .50x - .40x - 400$

$\boxed{P(x) = .10x - 400}$

6b) $P(500) = R(500) - C(500)$

Using the final $P(x)$ found:

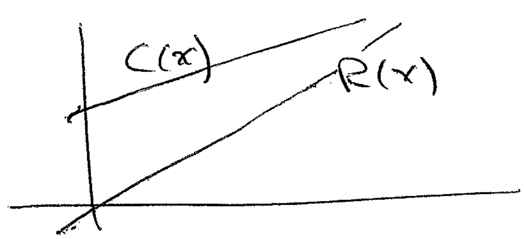
$P(500) = .10(500) - 400$

$= 50 - 400 = -350$

or \$350 loss

6c) Find x so $P(x) = 0$

This is the breakeven pt.



$P(x) = 0 = .10x - 400$

$400 = .10x$

$\boxed{x = 4000 \text{ copies}}$

7. a) Given fixed \$150
and $C(10) = \$300$
find $C(x)$.

$$C(x) = mx + \text{fixed}$$
$$C(10) = \underbrace{m(10) + 150 = 300}_{\text{solve for } m}$$

$$10m + 150 = 300$$
$$10m = 150$$
$$m = 15$$

So $C(x) = 15x + 150$

b) Marginal cost (that is, slope of the linear cost fcn) = \$100

$$C(x) = 100x + \text{fixed}$$

Now, to find fixed, use the other information: $C(10) = \$2237$

$$C(10) = \underbrace{100(10) + \text{fixed} = 2237}_{\text{solve for fixed}}$$

$$1000 + \text{fixed} = 2237$$
$$\text{fixed} = 1237$$

So $\boxed{C(x) = 100x + 1237}$

8. Marg. cost in (7a) is the slope since the cost fcn. is linear.

$MC = \$15$

<u>Note</u> $C(x) = 15x + 150$

The plant is producing x items. Each item costs \$15 to produce but the plant has \$150 into the production at the start. For very large x (high production) the cost function would look very different, Marginal cost would drop, but that's for our consideration after we discover how calculus gets us this information.

9. marg cost : \$2.50 to produce one item
Total cost for 100 items: $C(100) = \$300$

$$C(100) = \underbrace{2.50x + \text{fixed} = 300}_{\text{Solve for fixed for } x = 100}$$

$$(2.50)(100) + \text{fixed} = 300$$
$$\text{fixed} = 300 - 250 = \boxed{50}$$

So, $C(x) = 2.50x + 50$

9b) Find x to produce + sell so $P(x) = 0$ (break even)

$$P(x) = R(x) - C(x)$$

$$0 = px - (2.5x + 50)$$

We're given the selling price per item is \$6. $p = 6$

$$0 = 6x - 2.5x - 50 = \underbrace{\boxed{3.5x - 50}}^{P(x)}$$

$$0 = 3.5x - 50$$

$$x = \frac{50}{3.5} = \frac{500}{35} \approx \boxed{15 \text{ items}}$$

☆ Always round up!

9c) Find x to produce + sell so $P(x) = \$500$

Using $P(x) = 3.5x - 50$ from (b)

$$500 = 3.5x - 50$$

$$\cancel{\$500}\ x = \frac{550}{3.5} = \frac{5500}{35}$$

$$\approx \boxed{158 \text{ items}}$$

10. Each unit sells for $p = 5x + 20$ cents
(By the way, p = price, not to be confused with uppercase P, profit)

Revenue $= px = (5x+20)x$

$$\boxed{R(x) = 5x^2 + 20x}$$

11. Variable cost per unit is .50
Fixed cost per month is \$10,000

Find the monthly cost fcn. $C(x)$

$C(x) = .50x + 10{,}000$

12. Dessert maker's cost $C(x) = 7x + 21$
" " revenue $R(x) = 14x$

These are expensive pastries, it seems. The baker is spending \$7 on each one to make it, has a low fixed cost of \$21, and charges twice for each one sold. Breaking even won't take too many sales (x).

$P(x) = 14x - (7x+21) = 7x - 21 = 0$

$$\boxed{x = 3 \text{ desserts}}$$

12b) Profit $P(x)$ at $x = 100$ desserts

$$P(100) = 7(100) - 21 = \$679$$

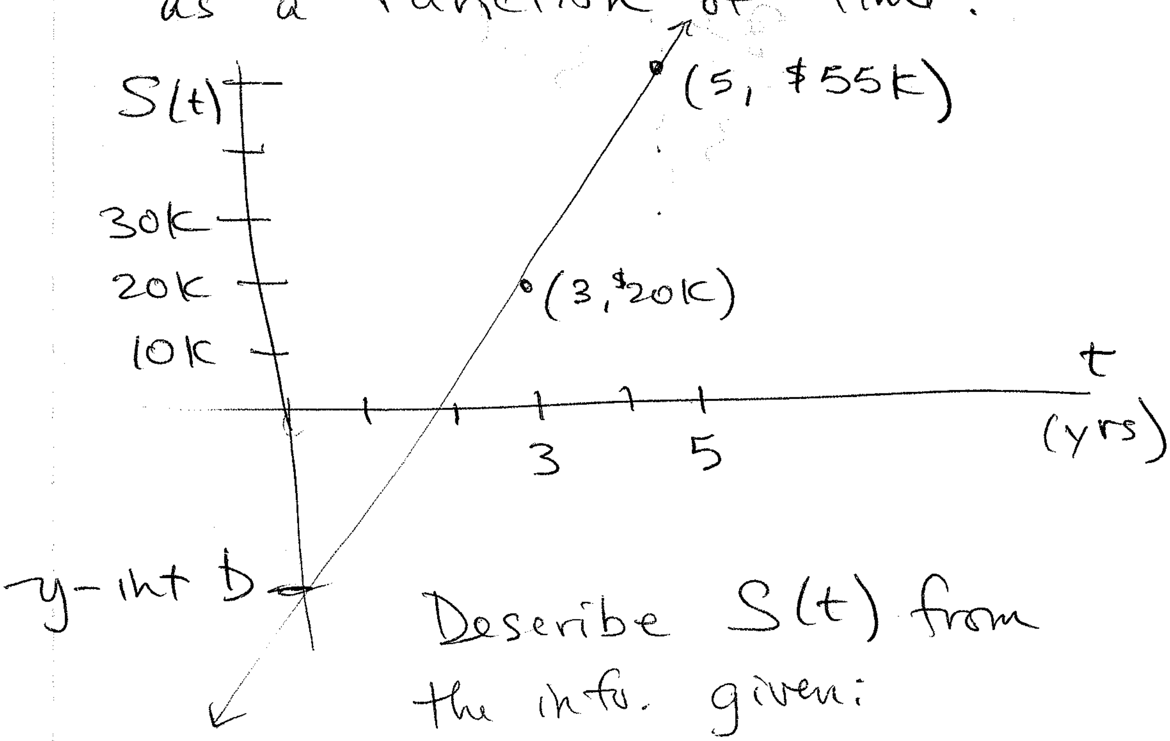
12c) Find x such that $P(x) = \$500$

It will be less than 100 (see (b))

$$500 = 7x - 21 \rightarrow x = \frac{521}{7}$$

$$x \approx 75 \text{ desserts}$$

13a) This one is a little different.
It's a linear equation problem in which sales are plotted as a function of time.



Describe $S(t)$ from the info. given:

The slope is sought; $m = \frac{\Delta S}{\Delta t}$

$$m = \frac{55K - 20K}{5 - 3} = \frac{\$35k}{2yr} = \frac{\$17.5k}{yr}$$

Sales increase at \$17,500 /yr

$$S(t) = 17,500t + b$$

← y-int

Just as we did with the first problems on finding eqns. of lines, here we seek the y-int.

We have 2 pts and the slope. Either pt. can be put into the pt - slope form of the line.

$$y - y_1 = m(x - x_1)$$

$$S - S_1 = 17,500(t - t_1)$$

Using $(t_1, S_1) = (3, 20K)$

$$S - \$20K = \$17.5K(t - 3)$$

$$S - 20,000 = 17,500t - 52,500$$

$$\boxed{S = 17,500t - 32,500}$$

Book uses x & y, which is fine.

13b) How long before $S(t)$ exceeds \$200,000 ?

We seek t in the equation:

$$S(t) = \underbrace{17{,}500\,t - 32{,}500 = 200{,}000}_{\text{Solve for } t}$$

$$17{,}500\,t = 232{,}500$$

$$t = \frac{232{,}500}{17{,}500} = \frac{2325}{175}$$

$$t \approx 13.28 \text{ yrs.}$$

What part of a year <u>in</u> <u>months</u> is .28 ? It's over $\frac{1}{4}$ (that is, .25)

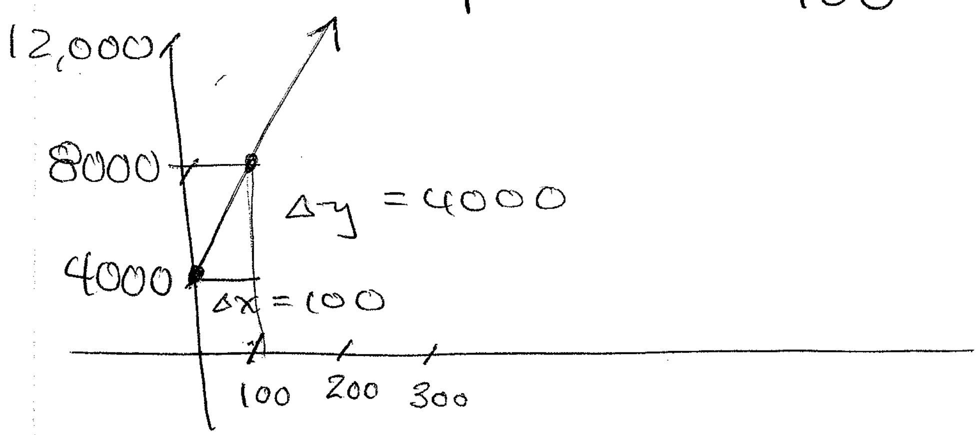
so $\approx \frac{1}{4}$ (12 months) $\approx$ 3 mos.

Answer: $\boxed{13 \text{ yrs } 3^{+} \text{ months}}$

14. $C(x) = 40x + 4{,}000$

The graph requires some scaling like in (13a). The x + $C(x)$ axes will differ in scale. With a slope of $40/1$ and a y-int of 4,000, we'll want y scale to be condensed compared to x.

$$m = \frac{40}{1} \longrightarrow \frac{4000}{100} = \frac{\Delta y}{\Delta x}$$



15. ~~16~~ $f(x)$ is miles as a function of gallons of gasoline, x.

The slope is the $\frac{\text{miles}}{\text{gal}}$ of 30 mpg.

So $f(x) = 30x$ is the number of miles you can go on x gallons.

The phrase "before you need more gas"

is misleading, since we aren't given that it started with a full tank. In this problem $f(x)$ = miles this car travels for each gallon of fuel.

$$f(x) = 30x, \quad f(10) = 300 \text{ miles}$$

$$f(12) = 360 \text{ miles, etc.}$$

$f(16.5)$ makes no sense because the fuel capacity is 15 gal. But on the other hand, it just ~~[illegible]~~ indicates miles possible with 16.5 gal. The car would have to fill up after 15 gal, then go another $(30)(1.5)$ miles.

$$f(16.5) = \underbrace{f(15)}_{\text{full tank}} + \underbrace{f(1.5)}_{\text{and some more}}$$

$$= (30)(15) + (30)(1.5)$$

$$= 450 + 45 = 495 \text{ mi}$$