

U-SUB

$$\rightarrow \text{ex. } \int \frac{-8}{x \ln 7x} dx = -8 \int \frac{1}{x} \cdot \frac{1}{\ln 7} \cdot dx \quad \begin{aligned} \text{let } u &= \ln 7 \\ du &= \frac{1}{x} dx \end{aligned}$$

$$= -8 \int \frac{1}{u} du = -8 \ln|u| + C \quad \boxed{-8 \ln|\ln 7| + C}$$

$$\rightarrow \text{ex. } \int \frac{\ln x}{x} dx = \int \ln x \cdot \frac{1}{x} dx \quad \begin{aligned} \text{let } u &= \ln x \\ du &= \frac{1}{x} dx \end{aligned}$$

$$- \int u du = \frac{u^2}{2} = \boxed{\frac{1}{2} \ln x^2 + C}$$

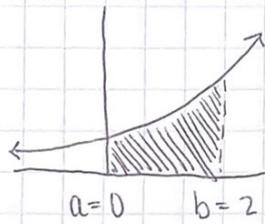
$$\rightarrow \int x^3 e^{x^4} dx \quad \begin{aligned} \text{let } u &= x^4 \Rightarrow \frac{1}{4} \int e^u du \\ du &= 4x^3 dx \end{aligned}$$

$$\frac{du}{4} = \frac{4x^3 dx}{4} \quad \boxed{f(x) = \frac{1}{4} e^{x^4} + C}$$

### FUNDAMENTAL THEOREM OF CALCULUS

→ If  $f$  is an anti-derivative of  $f(x)$ , a continuous function, then  $F' = f$ . In fact,  $F + C$  are anti-derivatives of  $f(x)$  because  $(F + C)' = F' + 0 = F' = f$

→ useful because, given rate function, we can find the area function/value on  $[a, b]$  the total value function " " the accumulation functions " "



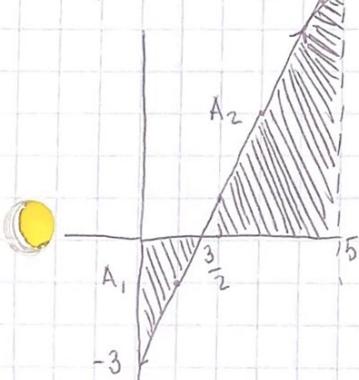
→ but the value of this accumulation function from  $x=0$  to  $x=2$  is a number

$$\int_0^2 e^x dx = e^x \Big|_0^2 = e^2 - 1$$

$$\int_0^2 f(x) dx = F(x) \Big|_0^2 = F(b) - F(a)$$

$$\rightarrow y = 2x - 3 \quad [0, 5]$$

$$\begin{aligned} A_1 &= \frac{3}{2} = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{2}{1} \\ A_2 &= \frac{49}{4} = \frac{1}{2} \cdot \frac{7}{2} \cdot 7 \end{aligned}$$



$$A_1 + A_2 = \frac{55}{4}$$

$$\rightarrow \text{ex. } \int_0^5 2x - 3 dx = x^2 - 3x \Big|_0^5 = (5)^2 - 3(5) = 10$$

\* To find area using integration, we need to recognize that if  $f(x)$  lies below the x-axis, then  $\int f(x) dx < 0$

$$\rightarrow \text{ex. } \int_0^2 -1 dx = -x \Big|_0^2 = -2$$

signed area of rectangle under x-axis on  $[0, 2]$

7. a.  $\int -32t \, dt = -\frac{32t^2}{2} = -16t^2 + C$

 $s(0) = 400$ 
 $s(0) = -16(0)^2 + C = 400 \rightarrow C = 400$ 
 $s(t) = -16t^2 + 400$ 
 $s(2) = -16(2)^2 + 400 = 336 \text{ ft.}$ 

b.  $s(t) = -16t^2 + 400 = 0$

 $\frac{400}{16} = t^2$ 
 $t = 5 \text{ sec.}$

8.  $\int 1.30 + 0.016x - 0.0018x^2 \, dx = 1.30x + \frac{0.016x^2}{2} + \frac{0.0018x^3}{3} = 1.3x + 0.03x^2 - 0.0006x^3 + C$

 $P(0) = 95 \text{ given}$ 
 $P(0) = 0 + 0 - 0 + C = 95 \rightarrow C = 95$ 
 $P(x) = 1.3x + 0.03x^2 - 0.0006x^3 + 95$

ANTIDERIVATIVE  $F(x)$  of  $f(x)$

- Such that  $F' = f$  (in fact,  $F = G + C$ )
- ex.  $s(t)$  is the antiderivative of  $v(t)$
- then, we find  $C$  for a given boundary condition by knowing a point on one of the  $F + C$ , the notation changes a bit
- ex. If  $f'(x) = 4x - 2$  and  $f(0) = -1 \rightarrow$  find antiderivative and solve for  $C$

$$\int f'(x) \, dx = \int (4x - 2) \, dx = 2x^2 - 2x + C$$

$$f(0) = 0 - 0 + C = -1 \rightarrow C = -1$$

$$f(x) = 2x^2 - 2x - 1$$

$$\int \frac{dx}{x} = \int x^{-1} = \int \frac{1}{x} \, dx$$

- Antiderivative = indefinite integral, but to use the operation of a-d (integration), we need the sense that the definite integral of  $f(x)$  on closed interval gives the area under the curve  $f(x)$   $[a, b]$
- The area gives total value of rate  $f(x)$  on  $[a, b]$ ; or, gives the accumulation under  $f$  on  $[a, b]$

$$\int v(t) \, dt = s(t)$$

The Fundamental Theorem of Calculus: If  $F$  is antiderivative of  $f(x)$  then  $\int_a^b f(t) \, dt = F(b) - F(a)$

so  $\frac{d}{dx} \int_a^x f(t) \, dt = f(x)$  and  $\int_a^b f(x) \, dx = F(b) - F(a) \leftarrow (c \text{ drops out}\right)$

Total Value Function

$$\text{find } \int \left(\frac{3}{x} + 2\right) \, dx, \text{ where } F(1) = 1$$

$$f(x) = 3\ln x + 2x + C$$

$$f(1) = 3\ln(1) + 2(1) + C = 1$$

$$= 0 + 2 + C = 1$$

$$C = -1$$

$$f(x) = 3\ln x + 2x - 1$$

Total Value Number

$$\text{find } \int_2^4 \left(\frac{3}{x} + 2\right) \, dx$$

$$3\ln x + 2x \Big|_2^4$$

$$F(4) - F(2) = 3\ln 4 + 2(4) - (3\ln 2 + 2(2)) =$$

$$3(\ln 4 - \ln 2) + 4 = 3\ln \frac{4}{2} + 4$$

$$= 3\ln 2 + 4$$

### CH31: V-SUBSTITUTION

→ theoretically the same as chain rule notation and theory

→ u-forms:  $u^n \rightarrow n u^{n-1}$ ,  $e^u \rightarrow e^u \cdot u'$ ,  $\ln u \rightarrow \frac{1}{u} \cdot u'$

→ consider that  $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$  ← chain rule

→ To apply this ideas to antiderivatives, we need to identify within the integral a composition (looking for  $\int u^n du$ ,  $\int e^u du$ ,  $\int \frac{1}{u} du$ )

→ u-sub example

$$\int (x+5)^2 dx \quad \left. \begin{array}{l} \text{let } u = x+5 \\ \frac{du}{dx} = 1 \rightarrow du = 1 dx \end{array} \right\} \text{KEY}$$

$$\int u^2 du = \frac{u^3}{3} + C \rightarrow \text{re-sub} \rightarrow \boxed{\frac{(x+5)^3}{3} + C}$$

$$\int (x^3 - 2)^2 x^2 dx \quad \left. \begin{array}{l} \text{let } u = x^3 - 2 \\ \frac{du}{dx} = 3x^2 \end{array} \right.$$

$$\int u^2 \frac{1}{3} du \quad \left. \begin{array}{l} du = 3x^2 dx \\ \frac{1}{3} du = x^2 dx \end{array} \right.$$

$$= \frac{1}{3} \cdot \frac{(x^3 - 2)^3}{3} + C$$

U-SUB OHM HW

$$\int x^3 (x^4 - 2)^8 dx \quad \left. \begin{array}{l} u = x^4 - 2 \\ \frac{du}{dx} = 4x^3 \end{array} \right.$$

$$\frac{1}{4} \int u^8 du = \frac{u^9}{9} + C \quad \left. \begin{array}{l} du = 4x^3 dx \\ \frac{1}{4} du = x^3 dx \end{array} \right.$$

$$\int x^3 e^{x^4} dx \quad \left. \begin{array}{l} u = x^4 \\ \frac{du}{dx} = 4x^3 \end{array} \right.$$

$$\frac{1}{4} \int u e^u du = \frac{1}{5} u^5 e^u + C$$

$$\int 5(t+2)^{-5} dt \quad \left. \begin{array}{l} u = t+2 \\ \frac{du}{dt} = 1 \end{array} \right.$$

$$\int \frac{du}{dx} dx = dt \quad \left. \begin{array}{l} u = 1x+7 \\ du = 1 dx \end{array} \right.$$

$$= \int \frac{1}{u} du = \ln|u| + C \quad \left. \begin{array}{l} u = 1x+7 \\ = \ln|x+7| + C \end{array} \right.^2$$

$$\ln|2+7|^2 - \ln|0+7|^2 \\ \ln\left(\frac{9}{7}\right)$$

$$\int \frac{2dx}{x \ln(5x)} = 2 \cdot \int \frac{1}{x \ln(5x)} dx$$

$$\left. \begin{array}{l} u = \ln(5x) \\ du = \frac{1}{5x} \cdot 5 \cdot dx \\ du = \frac{1}{x} dx \end{array} \right. \quad \begin{aligned} &= 2 \cdot \int \frac{1}{u} du \\ &= 2 \cdot \ln|u| + C \\ &= 2 \cdot \ln|\ln(5x)| + C \end{aligned}$$

$$\int x^3 \sqrt{14+x^4} dx = \frac{1}{4} \int u^{1/2} du = \frac{1}{4} \frac{u^{3/2}}{3/2}$$

$$\left. \begin{array}{l} u = 14+x^4 \\ du = 4x^3 dx \end{array} \right. \quad \begin{aligned} &= \frac{1}{4} \frac{(14+x^4)^{1/2}}{1/2} du \\ &= \frac{1}{4} (14+x^4)^{1/2} du \end{aligned}$$

$$\int \frac{1}{4} du = x^3 dx \quad \left. \begin{array}{l} u = x^4 + 6 \\ du = 4x^3 dx \end{array} \right.$$

$$\left. \begin{array}{l} \frac{1}{4} du = x^3 dx \\ u = x^4 + 6 \\ du = 4x^3 dx \\ \frac{1}{4} du = x^3 dx \end{array} \right. \quad \begin{aligned} &= \frac{1}{4} \int \frac{1}{u} du \\ &= \frac{1}{4} \ln(u) + C \\ &= \frac{1}{4} \ln(x^4 + 6) + C \end{aligned}$$