

For the function $f(x, y) = 6y^2 - 2y^3 + 3x^2 + 6xy$, write out all first and second order partial derivatives

$$f_x =$$

$$f_y =$$

$$f_{xx} =$$

$$f_{yy} =$$

$$f_{xy} =$$

Find the critical values (x_0, y_0) of the function, that is, those points where $f_x = 0$ and $f_y = 0$.

Solve the equation of the *discriminant* for *each* of the critical points you found:

$$D = f_{xx}f_{yy} - f_{xy}^2$$

Determine if these values constitute local maxima, minima, or saddle points using the criteria:

If $D < 0$ then (x_0, y_0) is a saddle point.

If $D > 0$ then (x_0, y_0) is $\begin{cases} \text{maximum if } f_{xx} < 0 \text{ \& } f_{yy} < 0 \\ \text{minimum if } f_{xx} > 0 \text{ \& } f_{yy} > 0 \end{cases}$

Finally, give the value of the function at the various extremes and/or saddle points.

Do the same for the function:

$$f(x, y) = 4x^2 + y^2 + 2x^2y - 1$$