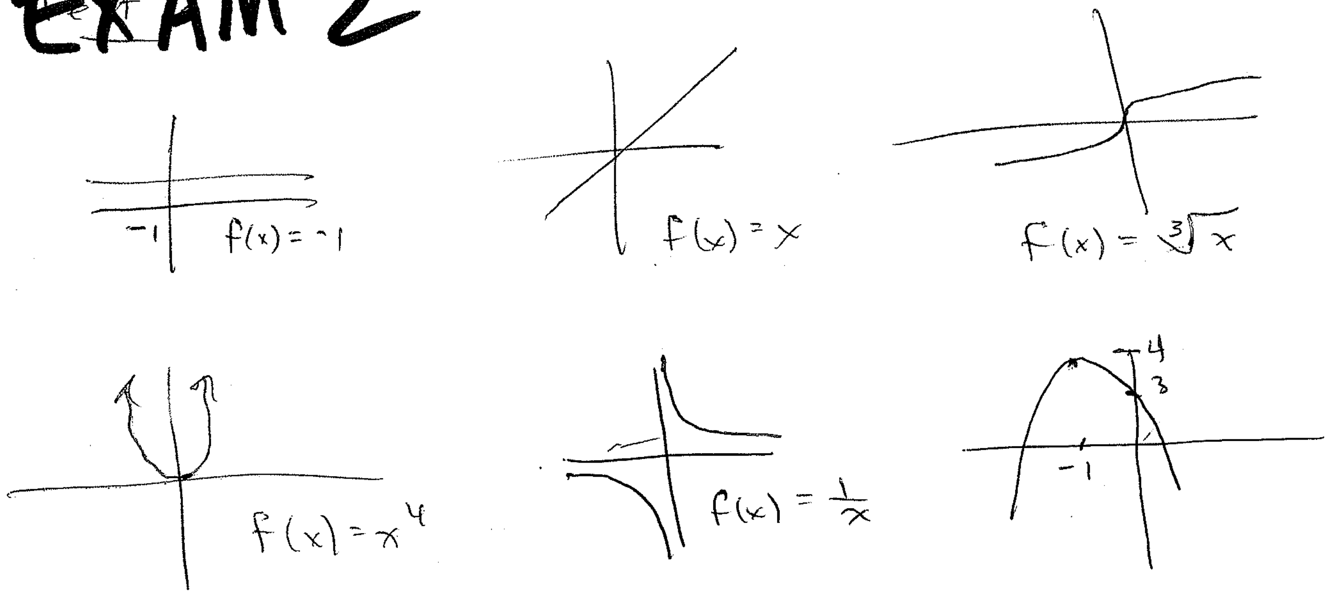
EXAM 2

1.



2. $f(x) = \sqrt{x}$ (1) shifts right 3 units, (2) vertically compresses by factor of 4, (3) reflects over y-axis

(1) $f_1(x) = \sqrt{x-3}$

(2) $f_2(x) = \frac{1}{4}\sqrt{x-3}$

(3) $f_3(x) = \frac{1}{4}\sqrt{-x-3}$

3.

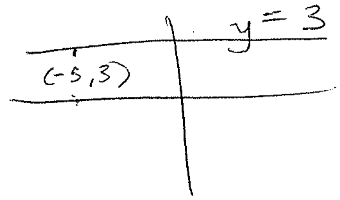
↑	↗	even, + l.c.
↙	↗	odd, + l.c.
↙	↘	even, − l.c.
↑	↘	odd, − l.c.

4. a) $y - y_1 = m(x - x_1)$, $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{y_1 - y_2}{x_1 - x_2}$

$\boxed{y - 3 = -\frac{3}{2}(x+5)}$ $m = \frac{-3-3}{-1 - -5} = \frac{-6}{4} = -\frac{3}{2}$

b) $\perp$ line: $m_\perp = -\frac{1}{m} = \frac{-1}{-3/2} = \boxed{\frac{2}{3} = m}$, $b = 4$ given, ~~$y = \frac{2}{3}x + 4$~~

$y = mx + b = \frac{2}{3}x + 4$

c) 

d) Midpt $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

simply the avg of each coordinate

$$\text{Mid.} = \left(\frac{-5 + -1}{2}, \frac{3 + -3}{2} \right)$$
$$= (-3, 0)$$

5. $f(x)$ has deg 4, $lc = -2$ } $\swarrow \searrow$ ends

$f(0) = 15$

$f(-1) = -2(-1)^4 - 3(-1)^3 + 15(-1)^2 + 31(-1) + 15$
$\uparrow$ root?

$= -2 + 3 + 15 - 31 + 15 = 0$ ✓ $x = -1$ is a root

So, $x + 1$ is a factor. Divide (synth. $\div$ fastest) into $P(x)$ to get first factorization.

$$\begin{array}{r|rrrrr} -1 & -2 & -3 & 15 & 31 & 15 \\ & & 2 & 1 & -16 & -15 \\ \hline & -2 & -1 & 16 & 15 & 0 \end{array}$$

root $\rightarrow$ -1 ; $\leftarrow$ coeffs.

$\hookrightarrow -2x^3 - x^2 + 16x + 15 \leftarrow$ factor.

Always try the same root, as a double root is possible

$$\begin{array}{r|rrrr} -1 & -2 & -1 & 16 & 15 \\ & & 2 & -1 & -15 \\ \hline & -2 & 1 & 15 & 0 \end{array}$$

$\hookrightarrow -2x^2 + x + 15$ is a factor.

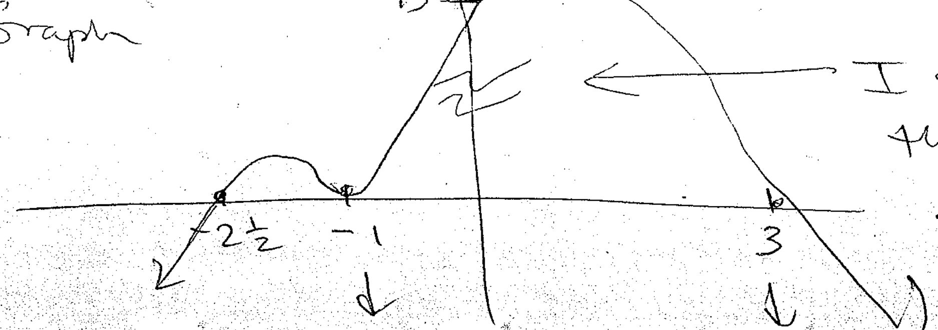
So far: $(x + 1)^2 (-2x^2 + x + 15)$

Factor by usual method the trinomial

$-(2x^2 - x - 15) = -(2x + 5)(x - 3)$

So, $P(x) = -(x + 1)^2 (2x + 5)(x - 3)$

5. Graph



I show a break this way, since y-axis isn't scaled to x-axis.

Mult. = 2 (even) Bounces!

M = 1 (odd) here & at $-2\frac{1}{2}$. These cross.

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6. a) $x(3x+2)=7$

$3x^2+2x-7=0 \longrightarrow$ QF $x=\dfrac{-2\pm\sqrt{4-4(3)(-7)}}{2(3)}$

$(3x\quad 7)(x\quad 1)$ – nope!

$=\dfrac{-2\pm\sqrt{88}}{6}=\dfrac{-2\pm2\sqrt{22}}{6}$

$=\boxed{\dfrac{-1\pm\sqrt{22}}{3}}$

b) $(x+2)^2=11 \Rightarrow \sqrt{(x+2)^2}=\pm\sqrt{11}$

$\rightarrow x+2=\pm\sqrt{11} \rightarrow$ ~~x=~~ $\boxed{x=-2\pm\sqrt{11}}$

c) $6x-11\sqrt{x}-10=0$ quadratic "in disguise". Substitute:

let $m=\sqrt{x}$, then $m^2=x$.

$6m^2-11m-10=0 \rightarrow (3m+2)(2m-5)=0$, trial & error

~~(6m+5)(m+2)~~ $\rightarrow 3m+2=0,\quad m=-2/3$

or $2m-5=0,\quad m=5/2$

Thus, $m=\sqrt{x}$

$m^2=x=(-2/3)^2$ or $(5/2)^2$

~~$(-2/3)^2=x$ or $(5/2)^2=x$~~

Discard, since $m=\sqrt{x}\neq$ negative

$\boxed{\text{Ans. } x=\dfrac{25}{4}}$

7) a) $f(x) = \dfrac{(2x-3)(x+1)}{(x+2)(x+1)} = \dfrac{2x-3}{x+2} \rightarrow \boxed{VA: x = -2}$ ← eqn.

$\rightarrow \boxed{\text{Hole at } x = -1}$

Deg P = Deg Q,
so HA = ratio of l.c. $= 2/1 = 2$ → eqn needed!

$\boxed{y = 2 \quad HA}$

$\boxed{\text{No slant asymptote } (\deg P \neq \deg Q + 1)}$

b) $g(x) = \dfrac{x^3 + 2x^2}{x^2 + x - 12} = \dfrac{x^2(x+2)}{(x-3)(x+4)} \rightarrow \boxed{VA: x = 3 \text{ \& } x = -4}$

Deg P > Deg Q by 1
so SA exists. It's quotient (w/o remainder)
of $\dfrac{x^3+2x^2}{x^2+x-12} \longrightarrow$

$$\begin{array}{r|l} & x + 1 \rightarrow SA \\ x^2 + x - 12 & x^3 + 2x^2 + 0 \\ & -(x^3 + x^2 - 12x) \\ & x^2 + 12x \\ & x^2 + x - 12 \\ \text{Rem.} & -x + 24 \end{array}$$

$\boxed{SA \quad y = x + 1}$

$\boxed{\text{No Holes (nothing cancelled)}}$

8. $f(x) = \dfrac{x^2 - 5x + 4}{(x+1)^3(x-2)} = \dfrac{(x-4)(x-1)}{(x+1)^3(x-2)}$

$\boxed{\text{Dom: } x \neq -1, 2}$

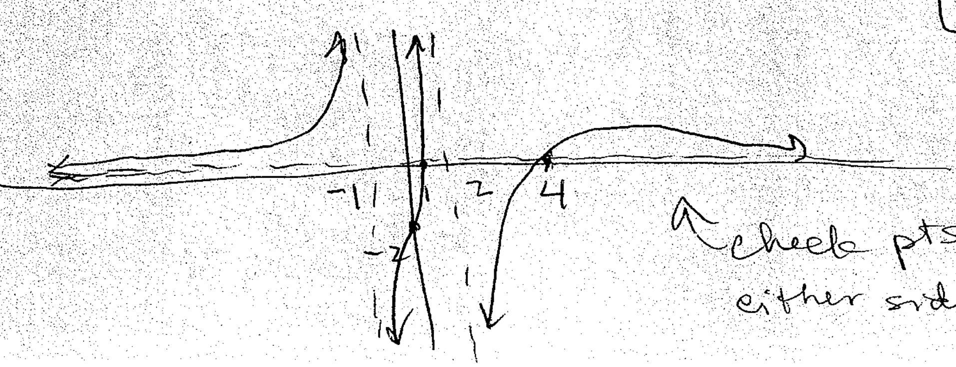
$\boxed{y\text{-int: } (0,-2)}$ $f(0) = \dfrac{4}{-2} = \boxed{-2}$

x-int: $0 = x^2 - 5x + 4 = (x-4)(x-1)$

$\boxed{x = 4, 1 \text{ x-intercepts}}$

$\boxed{\begin{array}{l} x = -1 \\ x = 2 \end{array} \quad VA}$

$\boxed{\text{Deg P < Deg Q, so HA is } y = 0}$



check pts either side of 4