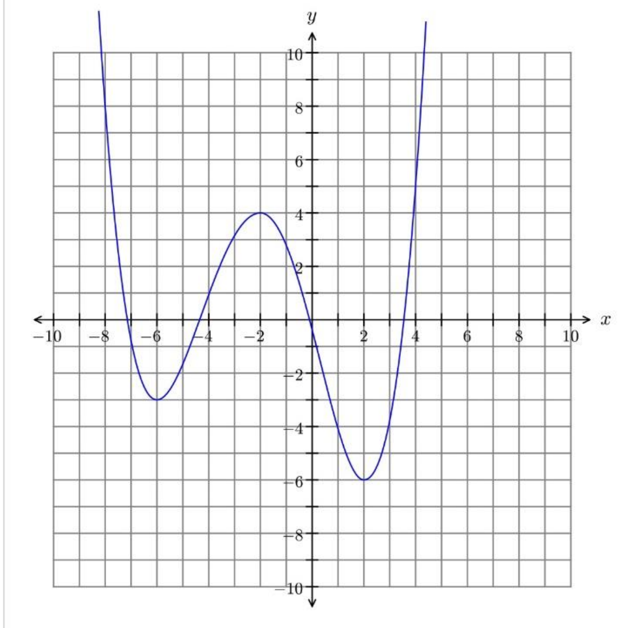
Review problems for Exam 2 Math 220

1. Give the *ordered pairs* of the extremes on intervals named. If feature is absent, write none:



On $(-\infty, \infty)$:

Local maxima:

Local minima:

Absolute maxima:

Absolute minima:

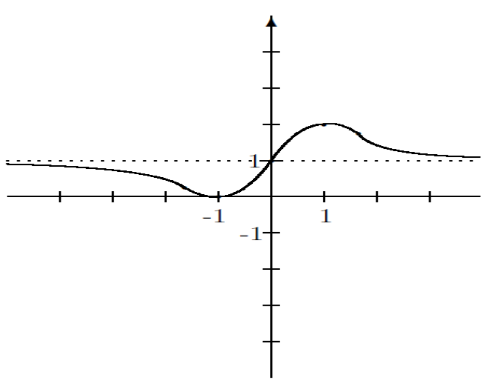
On $[-8, 2]$:

Local maxima:

Local minima:

Absolute maxima:

AbSolute minima:



On $(-\infty, \infty)$:

Local maxima:

Local minima:

Absolute maxima:

Absolute minima:

On $[0, \infty)$:

Local maxima:

Local minima:

Absolute maxima:

Absolute minima:

2. Find the equations of the lines tangent to the curve: $2e^x = y^2 - x$, at $x = 0$.

3. Given the curve $x^2 + 3y^2 = 22$:

   a) What two values of y does the curve attain when $x = 2$?

   b) Find the equation of the line tangent to the positive value of y that you found in (a). You must use implicit differentiation to find the slope.

4. The demand equation for a product shows us that the quantity produced varies with the price according to the equation $q = 1200/p$. The price is increasing at a rate of \$0.06 per month. How fast is the demand for this product changing when the price is \$6.00? Simplify to reduced form.

5. $R(x) = 50x - \frac{1}{2}x^2$; $C(x) = 4x + 10$. Revenue and cost are in dollars.

   a) Find the *rate* at which profit is changing when $x = 10$ and $dx/dt = 5$ units per day.

   b) Draw a graph of the profit function. At what value of x (level of sales) is profit at a maximum?

6. A pole 13 ft long leans against a vertical wall. If the lower end is moving away from the wall at the rate of 0.4 ft/sec, how fast is the upper end coming down when the lower end is 12 ft from the wall?

7. a) A pebble is dropped into a pool of water, generating circular ripples. The radius of the largest ripple is increasing at a constant rate of 6 inches per second. What is the increase in the area of the big ripple circle after 3 seconds have passed? (HINT: You will need to find the radius length at time 3 seconds).

   b) What is the increase in the circumference of the ripple after 3 seconds have passed?

8. Given the function $f(x) = 2x^3 - x^4$, answer each of the questions, showing all your work.
   Answer with interval notation for domain and other features.
   Write 'none' or 'nowhere' where appropriate.

   a) D_f:

   b) Intercepts:

   c) End behavior; that is, $\lim_{x \to -\infty} f(x) =$ ____ and $\lim_{x \to \infty} f(x) =$ _____

   d) $f'(x) =$ $f''(x) =$

   e) Critical numbers (x values only):

f) Use the number lines for the sign analysis:

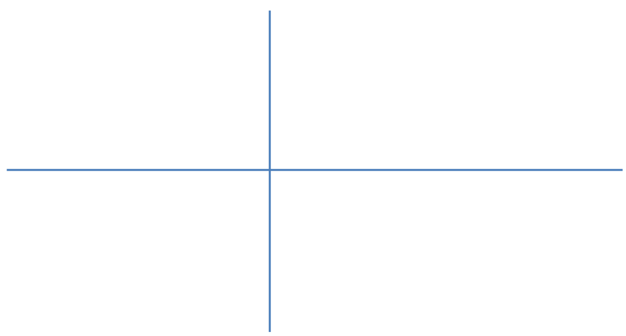
g) f increases on ____________________ f decreases on ____________________

h) Local maximum __________Local minimum ______________ (ordered pairs)

i) f is concave up on ___________________ f is concave down on __________________

i) f has a point of inflection at ________________ (ordered pair)

j) Sketch the function:



9. Consider a function, and its derivatives: $f(x)=\dfrac{x^2-1}{3x^2}$, $f'(x)=\dfrac{2}{3x^3}$, $f''(x)=\dfrac{-2}{x^4}$

a) What are the intercepts of $f(x)$, in ordered pair form?

b) What is the vertical asymptote?

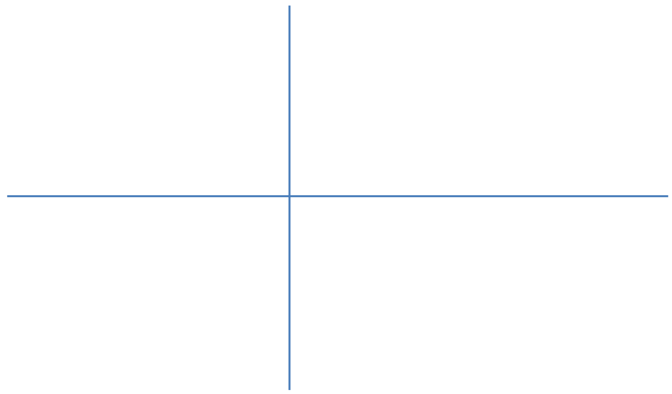
c) Are there any critical numbers? _________ Explain.

d) Are there any POI? _________ Explain.

e) $\lim\limits_{x\to\infty}\dfrac{x^2-1}{3x^2}$ is an indeterminate form, upon inspection. *Without using a shortcut*, find this limit.

f) Hence, what is the horizontal asymptote?

g) Sketch a graph of this function with its features clearly marked:



10. a) The intermediate value theorem states that if a function f is continuous on a closed interval $[a, b]$ and if the sign of f changes (say positive to negative or vice versa) on $[a, b]$, then it has at least one real root on (a, b).

Verify that $f(x) = x^4 - 7x^3 + 4x - 1$ has at least one root between $x = -1$ and $x = 1$?

b) What theorem ensures that a continuous function on a closed interval is guaranteed to have an absolute maximum and an absolute minimum?

c) Draw a secant line from (0, 1) to (3, –2). Find the slope of this line? Then draw a tangent line to the graph that illustrates the mean value theorem. Thus, at the point of tangency $f'(x) =$ ___ ?