

No calculators or other devices. Phones off and out of sight. No smart watches. Time: 90 min.

Page	1	2	3	4	Exam pts	Course pts
Points	25	22	25	25	100	200
Score						

1. Find the derivatives. Eliminate negative exponents, reduce numeric terms, combine like terms.

5
a) $f(x) = \frac{x^3}{4} + \frac{6}{x^2} - \sqrt[3]{x^2 + 1}$ $f'(x) = \frac{3x^2}{4} - \frac{12}{x^3} - \frac{2x}{(x^2+1)^{2/3}}$

(+1) (+1) (+1)

6
b) $g(x) = 3^{2x} + 2\log(1-x)$ $g'(x) = 3^{2x} \cdot \ln 3 \cdot 2 + \frac{-2}{(1-x)\ln 10}$

(+1) (+1) (+1) (+1) (+1)

5
c) $f(x) = \sqrt{\frac{x+1}{8x}}$ $f'(x) = \frac{1}{2} \left(\frac{x+1}{8x}\right)^{-1/2} \left[\frac{1 \cdot 8x - (x+1)8}{(8x)^2} \right]$

(+1) power (+1) combine
(+1) chain (+1) neg exp
(+1) quotient

= $\frac{1}{2} \left(\frac{8x}{x+1}\right)^{1/2} \left(\frac{-1}{64x^2}\right)$
= $-\frac{1}{2} \sqrt{\frac{8x}{x+1}} \left(\frac{1}{64x^2}\right)$

d) $h(x) = e^{2x} + x^{-1} - x^6$

3
 $h'(x) = 2e^{2x} - \frac{1}{x^2} - 6x^5$ (+1) each

3
 $h''(x) = 4e^{2x} + \frac{2}{x^3} - 42x^6$ (+1) each

3
 $h^{(7)} = 2^7 e^{2x} - \frac{8 \cdot 7 \dots 2}{x^8} - 0$ = $128e^{2x} - \frac{40,320}{x^8}$

(+1) (+1) (+1) (+1) (+1)

2. Consider the functions on the specified domains. Identify values of x where the function attains a local extreme, and if it is a local max or local min. If there is no local extreme, write "NONE." Justify each with a sketch.

	Max or min	Sketch
4	(a) $y = 4 - x^2$ on $(-\infty, \infty)$ $x = 0$ max (+2)	
4	(b) $y = 5$ on $(-2, 2)$ $x = (-2, 2)$ all $x \in I$ are both max, min (+2)	
4	(c) $y = e^{-x}$ on $(-\infty, \infty)$ $x = \text{none}$ (+2)	

3. Wolfe Electronics determines the cost to produce auto antitheft devices is modeled by $C(x) = (3x + 4)^{1.5} + 30$, $0 \leq x \leq 50$, where x represents hundreds of devices produced and $C(x)$ represents production costs in thousands of dollars.

- (a) Determine the marginal cost function.

5.

$$C'(x) = 1.5(3x+4)^{0.5} \cdot 3 = 4.5(3x+4)^{0.5}$$

(+1) (+1) (+1) (+1)

- (b) Evaluate and interpret $C'(7)$.

8.

$$C'(7) = 4.5(25)^{0.5} = 4.5(25)^{1/2} = 4.5(5) = 22.5$$

(+1) (+1) (+1) (+1) (+1)

$C'(7) = 22.5$ which means that

production of the 800th device costs \$22,500
more than the 700th

(+2)

4. Find the critical numbers of each of the functions. For each x you find, name which of the two criteria that qualifies it as critical.

(a) $f(x) = 3x^2 + 2x^3$ (2)

$$f'(x) = 6x + 6x^2 = 6x(1+x) = 0 \quad \text{at } x=0, -1$$

$c=0, 1$, critical numbers where $f'(c)=0$ (1)

(b) $g(x) = \frac{4x}{x^2+1}$

$$g'(x) = \frac{4(x^2+1) - 4x(2x)}{(x^2+1)^2} = \frac{4 - 4x^2}{(x^2+1)^2} = \frac{4(1-x^2)}{(x^2+1)^2}$$

(1) $4(1-x^2) = 4(1-x)(1+x) = 0$

(2) for extra c.n., since $g'(c)=0$ (1)

(c) $h(x) = \sqrt[3]{x-2} = (x-2)^{1/3}$

$$h'(x) = \frac{1}{3(x-2)^{2/3}} \quad 3(x-2) = 0 \quad \text{at } x=2 \quad (1)$$

(2) since $h'(2)$ DNE (2)

$h' = 0$ nowhere (1)

10 5. Find the equation of line tangent to $xy + y^2 - 2x = 0$, at $x=1$, lying in the fourth quadrant.

$$y + x \frac{dy}{dx} + 2y \frac{dy}{dx} - 2 = 0$$

(4) $\frac{dy}{dx}(x+2y) = 2-y$

(1) $\frac{dy}{dx} = \frac{2-y}{x+2y}$

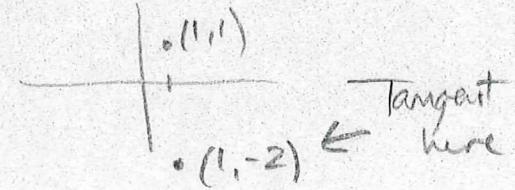
When $x=1$,
 $y+y^2-2=0$
 $y^2+y-2=0$

$$(y+2)(y-1)=0$$

$y=1, -2$ (1) for choice

In Q4, $\frac{dy}{dx} = \frac{2-(-2)}{1+2(-2)} = \frac{4}{-3}$

$| y+2 = \frac{-4}{3}(x-1) |$ (1)



6. (a) The demand equation for a product shows demand varies with the price according to the equation $q = \frac{1200}{p}$. The price is increasing at a rate of \$0.06 per month. How fast is demand for this product changing when price is \$6.00?

$$\frac{dq}{dt} = \frac{dq}{dp} \cdot \frac{dp}{dt} = -\frac{1200}{p^2} \cdot \frac{dp}{dt}$$

$$\begin{array}{r} 1200 \\ .06 \\ \hline 72.00 \end{array}$$

(+2)

(+2)
Scheme
correct

$$= -\frac{1200}{6^2} \cdot \frac{.06}{\text{mo}} \quad (+2) \text{ sub.}$$

$$= -\frac{72}{36} = -2 \frac{\text{units}}{\text{month}}$$

(+1) final

amount
demand
goes down

(+1) units

15

(b) A restaurant supplier services the restaurants in a circular area in such a way that the radius r is increasing at the rate of 2 mi per year at the moment when $r = 5$ mi. At that moment, how fast is area increasing?

$$\frac{dr}{dt} = 2, \text{ find } \frac{dA}{dt}, \text{ when } r = 5$$

(+1)

$$(+) A = \pi r^2$$

$$(+) \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} \quad (+2) \text{ for } \frac{dA}{dr} = 2\pi r$$

10

$$(+) \frac{dA}{dt} = 2\pi \cdot 5 \text{ mi} \cdot \frac{2 \text{ mi}}{\text{yr}} = 20\pi \frac{\text{mi}^2}{\text{yr}}$$

(+1)

sub