EXAM 1

Math 108-01-04 Fall 2015 Test 1 Solution Guide

#1a) $\dfrac{28x^2y^{-3}}{21x^{-2}y^4} = \dfrac{28x^2x^2}{21y^4y^3} = \dfrac{7x^4}{3y^7}$

b) $3(x^3y^5)^4 = 3x^{3\cdot4}y^{5\cdot4} = 3x^{12}y^{20}$

#2a) $\sqrt{24a^9b^6} = \sqrt{4\cdot6\cdot a^8\cdot a\cdot b^6} = 2a^4b^3\sqrt{6a}$

b) $2\sqrt{32x^2y^3} - xy\sqrt{98y} = 2\sqrt{16\cdot2\cdot x^2y^2y} - xy\sqrt{49\cdot2\cdot y}$

$= \underbrace{2\cdot4}_{8}\cdot x\cdot y\sqrt{2y} - 7xy\sqrt{2y} = xy\sqrt{2y}$

c) $\sqrt[3]{4a^2b^3}\cdot\sqrt[3]{8ab^5} = \sqrt[3]{32a^3b^8} = \sqrt[3]{8\cdot4\cdot a^3b^6\cdot b^2}$

$= 2\cdot a\, b^2\sqrt[3]{4b^2}$

#3) a) $16x^3-60x^2+50x = 2x(8x^2-30x+25)$

$= 2x(4x-5)(2x-5)$

b) $2x^2(x+1)^3 + x^4(x+1)^2$ ~~$= (x+1)(x^2)(x+1)^2 + x+1$~~

$= x^2(x+1)^2\left[2(x+1)+x^2\right] = x^2(x+1)^2\left(2x+2+x^2\right)$
x^2+2x+2

$= x^2(x+1)^2(x^2+2x+2)$

c) $16x^4-81 = (4x^2-9)(4x^2+9)$

$= (2x-3)(2x+3)(4x^2+9)$

4a) $x(x+4)=45$, $x^2+4x-45=0$ $(x-5)(x+9)=0$ $\boxed{x=5,-9}$

b) $\dfrac{6}{x-7}=\dfrac{8}{x-6} \rightarrow 6x-36=8x-56$

$20=2x \longrightarrow \boxed{x=10}$

c) $\left(\sqrt{2x+1}\right)^2=\left(2+\sqrt{x-3}\right)^2$

$2x+\cancel{1}=\cancel{4}+4\sqrt{x-3}+x-\cancel{3}$

$x=4\sqrt{x-3} \longrightarrow x^2=16(x-3) \rightarrow x^2-16x+48=0$

$\rightarrow (x-12)(x-4)=0 \rightarrow x=12,4$

$x=12$ Chk $\sqrt{2(12)+1} \stackrel{?}{=} 2+\sqrt{12-3} \longrightarrow 5=2+3$ ✓

$x=4$ Chk $\sqrt{2(4)+1} \stackrel{?}{=} 2+\sqrt{4-3} \longrightarrow \sqrt{9}=2+\sqrt{1} \rightarrow 3\stackrel{\checkmark}{=}3$

~~[illegible]~~

5.

$$
\begin{array}{r}
2x^2+x+9 \\
\boxed{x^2}-3x+1 \,\big)\, \boxed{2x^4}-5x^3+6x^2+0x-10 \\
-(2x^4-6x^3) \\
x^3+6x^2+0x \\
-(x^3-3x^2+1x) \\
9x^2-x-10 \\
-(9x^2-27x+9) \\
26x-19
\end{array}
$$

5.

$$
\begin{array}{r}
2x^2 + x + 7 + \frac{20x-17}{x^2-3x+1} \\
x^2 - 3x + 1 \overline{) \; 2x^4 - 5x^3 + 6x^2 + 0x - 10} \\
-(2x^4 - 6x^3 + 2x^2) \\
\hline
x^3 + 4x^2 + 0x \\
-(x^3 - 3x^2 + 1x) \\
\hline
7x^2 - x - 10 \\
-(7x^2 - 21x + 7) \\
\hline
20x - 17
\end{array}
$$

6. $2x^2 - 28x + 99 = 2(x^2 - 14x \quad) + 99$

$$= \underbrace{2\left(x^2 - 14x + \left(\tfrac{-14}{2}\right)^2\right) - 98}_{+98 \; = 0} + 99$$

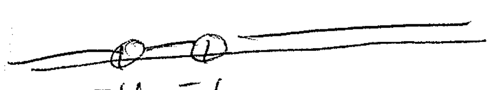
$$= \boxed{2(x-7)^2 + 1}$$

7. a) $f(x) = \begin{cases} x^2 + 3x + 2 & \text{if } x \geq 5 \\ \sqrt{6-x} & \text{if } x < -3 \end{cases}$ Dom.

$(-\infty, -3) \cup [5, \infty)$

b) $g(x) = \dfrac{(x-3)\cancel{(x+1)}}{(x+4)\cancel{(x+1)}}$ Dom $x \neq -4$ & -1

still must exclude from Dom.



$(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

c) $h(x) = \sqrt[10]{2-x}$ even root, requires $2 - x \geq 0$

so $x \leq 2$

or $(-\infty, 2]$

8. • $(f \circ g)(x) = f(g(x)) = f(x^2+3) = \frac{x^2+3}{x^2+3-4} = \frac{x^2+3}{x^2-1}$

• Dom: $x \neq \pm 1$

• Call $f \circ g$ "$h(x)$". Chk $h(x)$ against $h(-x)$

$h(-x) = \frac{(-x)^2+3}{(-x)^2-1} = \frac{x^2+3}{x^2-1}$ which $= h(x)$

This is definition of an even fcn.

• y-int: $h(0) = \frac{0^2+3}{0^2-1} = \boxed{-3}$

• $(g-f)(x) = x^2+3 - \frac{x}{x-4}$; $(g-f)(5) = 5^2+3-\frac{5}{5-4} = \boxed{23}$

• Is f one-one? i.e., If $f(a) = f(b)$ implies $a = b$, then f is one-one.

~~$f(a) = \frac{a}{a-4}$~~

So, assume $f(a) = f(b)$, that is $\frac{a}{a-4} = \frac{b}{b-4}$

Cross ⊗ : $a(b-4) = b(a-4)$

$\cancel{ab} - 4a = \cancel{ba} - 4b$

$-\cancel{4}a = -\cancel{4}b$

$\boxed{a = b}$ which is what we needed to show

$f(x) = \dfrac{x}{x-4}$ $\qquad f^{-1}(x) = ?$

$$y = \frac{x}{x-4} \longrightarrow x = \frac{y}{y-4} \longrightarrow xy - 4x = y$$

$$\longrightarrow xy - y = 4x \longrightarrow y(x-1) = 4x \Rightarrow y = \frac{4x}{x-1}$$

$$\longrightarrow f^{-1}(x) = \frac{4x}{x-1}$$

$$\frac{g(x+h) - g(x)}{h} = \frac{(x+h)^2 + 3 - (x^2+3)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3} - \cancel{x^2} - \cancel{3}}{h} = \frac{2x\cancel{h} + h^{\cancel{2}}}{\cancel{h}} = \boxed{2x+h}$$

9a) $2\displaystyle\sum_{i=4}^{9} i^2 = 2(4^2 + 5^2 + \dots + 9^2) = ?$

$$= 2(16 + 25 + 36 + 49 + 64 + 81) = 542$$

or $2\left[\displaystyle\sum_{i=1}^{9} i^2 - \sum_{i=1}^{3} i^2\right] = 2\left[\dfrac{9(9+1)(18+1)}{6} - \dfrac{3(3+1)(6+1)}{6}\right]$

$$= \cancel{2}\left(\frac{1710 - 84}{\cancel{6}\,3}\right) = \frac{1626}{3} = 542$$

I think the first way is better :)

b) $\displaystyle\sum_{i=7}^{11} \frac{i+2}{i^5} = \sum_{i=4}^{8} \frac{(i+3)+2}{(i+3)^5}$

$+3$

-3

c) $\sum_{i=1}^{50} (i+2)(i+3) = \sum_{i=1}^{50} (i^2 + 5i + 6)$

$= \sum_{i=1}^{50} i^2 + 5\sum_{i=1}^{50} i + \sum_{i=1}^{50} 6$

$= \frac{50(50+1)(100+1)}{6} + \frac{5\cdot(50)(50+1)}{2} + 50(6)$ etc