## Math 220: Exam 1

## Name

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## Spring 2019

## Instructor

| Page | $\mathbf{1}$ | 2 | 3 | 4 | Total | Course <br> Points |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Points | 20 | 33 | 22 | 25 | $\mathbf{1 0 0}$ | 200 |
| Score |  |  |  |  |  |  |

- Calculators are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- There are problems on the front of all pages. If you need more space, use the backs. You are not permitted to use any scrap paper on this exam.
- You may not use L'Hopital's Rule to evaluate limits on this exam, nor any exam in this course.
- You will have exactly 60 minutes to complete this exam.

1. (12 points) Find the domain of each of the following functions. Express answers using interval notation.
a) $f(x)= \begin{cases}\frac{1}{x-1} & \text { if } \mathrm{x} \leq-2 \\ 3^{x}+4 & \text { if } \mathrm{x}>-1\end{cases}$
$\operatorname{Dom}(f)=$
b) $g(x)=\frac{\sqrt{x+4}}{x^{2}-x-6}$
$\operatorname{Dom}(g)=$
c) $h(x)=\ln \left(x^{2}-9\right)$
$\operatorname{Dom}(h)=$
2. (8 points) Find the roots of each function below.
a) $f(x)=2 x^{4}-6 x^{3}-10 x^{2}$

Roots of $f$ :
b) $g(x)=3^{x-2}-9$

Roots of $g$ :
3. (33 points) Evaluate each of the following limits. If a certain limit does not exist, write "DNE."
a) $\lim _{x \rightarrow 5}\left(x^{3}-\frac{2 x}{x+5}+\log _{5} x-2^{x}\right)$
b) $\lim _{x \rightarrow 3} \frac{2 x^{2}-5 x-3}{x^{2}+x-12}$
c) $\lim _{x \rightarrow 2} \frac{\sqrt{3 x+10}-4}{x^{2}-2 x}$
d) $\lim _{x \rightarrow-4^{-}} \frac{x^{2}-x-20}{x+4}$
e) Suppose $f(x)= \begin{cases}3 x+1 & \text { if } \mathrm{x}<2 \\ 5 & \text { if } \mathrm{x}=2 \\ \log _{2}(x+6)+4 & \text { if } \mathrm{x}>2\end{cases}$
$\lim _{x \rightarrow 2^{-}} f(x)=$
$\lim _{x \rightarrow 2^{+}} f(x)=$ $\qquad$

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\lim _{x \rightarrow 2} f(x)=
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f) In part e) above, is $f$ continuous at $x=2$ ? Fully justify your answer.
4. (22 points) An object moves horizontally along the number line. Its position (measured in meters) after $t$ minutes is given by the function $s(t)=3 t^{2}-12 t+1$ for all $0 \leq t \leq 10$.
a) Find the average velocity of the object during the first six minutes of its movement. Express your answer with appropriate units.
b) Find $s^{\prime}(t)$ using the definition of derivative.
c) Use your answer from part b) to find the instantaneous velocity of the object when $t=5$. Express your answer with appropriate units.
d) Write the equation of the line tangent to $s(t)$ at the point where $t=1$. Any form of linear equation is acceptable.
e) Is the object always moving forward along the number line? Justify your answer.
5. (14 points) Answer each of the following questions involving money growth. You may leave your answers unsimplified, but they should be numerical.
a) Camille invests $\$ 3000$ into an investment that earns $10 \%$ annual interest, compounded continuously. How long will it be until the investment is worth $\$ 12,000$ ?
b) Dylan has $\$ 2000$ right now and wants to have $\$ 10,000$ in 20 years. A bank is offering Dylan a savings account where interest is compounded three times annually. Under what condition should Dylan invest his money in this account in order to achieve his goal?
6. (11 points) Jared starts an online business to sell hats. His fixed monthly cost is $\$ 600$, and it costs him \$9 to make each hat. Currently he sells each hat for $\$ 15$.
a) Write Jared's monthly profit function $P(x)$, where $x$ represents the number of hats that Jared makes and sells.
b) How many hats does Jared need to sell in order to break even?
c) Suppose Jared wants to make a profit of $\$ 1000$ in a given month. Write an expression for $p$, the price at which he sells each hat, as a function of $x$, the number of hats sold.

