Derivative Rules

So-called 'x-forms' apply when $y = f(x)$; so-called 'u-forms' apply when $y = f(u(x))$, hence the chain rule is applied: $y' = f'(u(x))u'(x)$.

1. Power rule, x-form: $y = x^n, \quad y' = nx^{n-1}$ where $n \in \mathbb{R}$

   Memorize the cases: $y = c, \quad y' = 0$; $\quad y = x, \quad y' = 1$; $\quad y = \frac{1}{x}, \quad y' = -\frac{1}{x^2}$; $\quad y = \sqrt{x}, \quad y' = \frac{1}{2\sqrt{x}}$

   Power rule, u-form: $y = u^n$, where $u \equiv u(x)$ $\quad y' = nu^{n-1} \cdot u'$ or $\frac{dy}{dx} = nu^{n-1} \cdot \frac{du}{dx}$

2. Exponential rule, x-form, base e: $y = e^x, \quad y' = e^x$

   Exponential rule, x-form, base a: $y = a^x, \quad y' = a^x \ln a$

   Exponential rule, u-form, base e: $y = e^u, \quad y' = e^u \cdot u'$ or $\frac{dy}{dx} = e^u \cdot \frac{du}{dx}$

   Exponential rule, u-form, base a: $y = a^u, \quad y' = a^u \cdot \ln a \cdot u'$ or $\frac{dy}{dx} = a^u \cdot \ln a \cdot \frac{du}{dx}$

   Memorize the common derivative of $y = e^{kx}, \quad y' = ke^{kx}$ for example: $y = e^{-x}, y' = -e^{-x}$

3. Logarithm rule, x-form, base e: $y = \ln x, \quad y' = \frac{1}{x}$

   Logarithm rule, u-form, base e: $y = \ln u, y' = \frac{1}{u} \cdot u'$ or $\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$

   Logarithm rule, x-form, base a: $y = \log_a x, \quad y' = \frac{1}{x \cdot \ln a}$

   Logarithm rule, x-form, base a: $y = \log_a u, \quad y' = \frac{1}{u \cdot \ln a} \cdot u'$ or $\frac{dy}{dx} = \frac{1}{u \cdot \ln a} \cdot \frac{du}{dx}$

Product rule: $y = f(x)g(x)$, $y' = f'(x)\cdot g(x) + g'(x)\cdot f(x)$

Chain rule: $y = f(u(x))$, $y' = f'(u(x))\cdot u'(x)$

Quotient rule: $y = \frac{f(x)}{g(x)}$, $y' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

Any expression within a function whose derivative requires the product or quotient rule will follow one of the *x*-forms above (power, exp, log) and may entail use of the chain rule, the *u*-forms. See the plentiful examples in the book and supplementary materials.

Examples—notice when the coefficient goes along for the ride

Simple power rule: $y = 0.09x^2$, $y' = 0.18x$; $y = x^{\sqrt{7}}$, $y' = \sqrt{7}x^{\sqrt{7}-1}$; $y = \frac{5}{x^4} = 5x^{-4}$, $y' = -20x^{-5}$

Power rule with a polynomial: $y = (6-2x^4)^3$, $y' = 3(6-2x^4)^3(-8x)$

Power rule with a radical: $y = (3+\sqrt{x})^6$, $y' = 6(3+\sqrt{x})^5 \cdot \frac{1}{2\sqrt{x}}$

Power rule with log: $y = (\ln x)^3$, $y' = 3(\ln x)^2\left(\frac{1}{x}\right)$

Exponential rule: $y = 0.1e^{-5x}$, $y' = -0.5e^{-5x}$; $y = 4\cdot 2^x$, $y' = 4\cdot 2^x \cdot \ln 2$

Log rule: $y = \ln(15x)$, $y' = \frac{1}{15x}\cdot 15 = \frac{1}{x}$

Notice that if $y = \ln(cx)$, then $y' = \frac{1}{cx}\cdot c = \frac{1}{x}$. This is because $\ln(cx) = \ln c + \ln x$, so $\frac{d}{dx}[\ln(cx)] = \frac{d}{dx}[\ln c] + \frac{d}{dx}[\ln x] = 0 + \frac{1}{x} = \frac{1}{x}$

$y = \log(12x)$, $y' = \frac{1}{12x\cdot\ln 10}\cdot 12 = \frac{1}{x\cdot\ln 10}$; $y = \log_3(x^2+1)$, $y' = \frac{1}{(x^2+1)\ln 3}\cdot 2x$