MAT 270 - Derivative Practice I

Find the derivative of each of the following functions and simplify.

1. $f(x) = 4x^3 - 3x^2 + 2x - \pi$

$f'(x) = 12x^2 - 6x + 2$

rewrite 2. $f(x) = \frac{x^2}{3} - \frac{3}{x^2}$ $= f(x) = \frac{x^2}{3} - 3x^{-2}$

$f'(x) = \frac{2}{3}x + 6x^{-3}$

distribute first $f(x) = -3(2x^2 - 5x + 1)$ $= -6x^2 + 15x - 3$

$f'(x) = -12x + 15$

rewrite 4. $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$ $= x^{1/2} - x^{-1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$

quotient 5. $f(x) = \frac{x+1}{x-2}$

$f'(x) = \frac{(x-2)(1) - (x+1)(1)}{(x-2)^2} = \frac{(x-2) - (x+1)}{(x-2)^2} = \frac{-3}{(x-2)^2}$

rewrite OR quotient 6. $f(x) = \frac{x^2 - 2}{x^2}$ $= \frac{x^2}{x^2} - \frac{2}{x^2} = 1 - \frac{2}{x^2} = 1 - 2x^{-2}$

$f'(x) = 4x^{-3}$

quotient 7. $f(x) = \frac{x^2}{x^2 - 2}$

$f'(x) = \frac{(x^2-2)(2x) - (x^2)(2x)}{(x^2-2)^2} = \frac{(x^2-2)(2x) - 2x^3}{(x^2-2)^2}$

product rule OR rewrite 8. $f(x) = \sqrt{x}(x^2 + 1)$

$= x^{1/2}(x^2+1)$ $f'(x) = x^{1/2}(2x) + (x^2+1)\frac{1}{2}x^{-1/2}$

9. $f(x) = \frac{e^x}{e^x - 1}$

quotient

$f'(x) = \frac{(e^x-1)e^x - e^x(e^x)}{(e^x-1)^2} = \frac{e^{2x} - e^x - e^{2x}}{(e^x-1)^2} = \frac{-e^x}{(e^x-1)^2}$

rewrite 10. $f(x) = \frac{2}{\sqrt{x}} + \frac{\sqrt{x}}{2}$

$f(x) = 2x^{-1/2} + \frac{1}{2}x^{1/2}$

$f'(x) = -x^{-3/2} + \frac{1}{4}x^{-1/2}$

quotient 11. $f(x)=\frac{2x}{x-1}$

$$f'(x)=\frac{(x-1)2-2x(1)}{(x-1)^2}=\frac{2x-2-2x}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

product 12. $f(x)=(3x-2)(2x+1)$

$$f'(x)=(3x-2)(2)-(2x+1)(3)=\cancel{6x}-4-\cancel{6x}-3=-7$$

rewrite 13. $y=5x^2-5\sqrt{x}-\frac{3}{x}$

$$y=5x^2-5x^{1/2}-3x^{-1}$$

$$y'=10x-\frac{5}{2}x^{-1/2}+3x^{-2}$$

quotient 14. $y=\frac{\sqrt{x}}{\sqrt{x}-1}=\frac{x^{1/2}}{x^{1/2}-1}$

$$y'=\frac{(x^{1/2}-1)\frac{1}{2}x^{-1/2}-x^{1/2}\left(\frac{1}{2}x^{-1/2}\right)}{(x^{1/2}-1)^2}$$

quotient 15. $y=\frac{e^x}{x}$

$$y'=\frac{xe^x-e^x}{x^2}=\frac{e^x(x-1)}{x^2}$$

16. $y=6x^{\frac{-3}{2}}+7x^{\frac{1}{5}}+1$

$$y'=-9x^{-5/2}+\frac{7}{5}x^{-4/5}$$

quotient 17. $y=\frac{-7}{1-x^3}$

$$\frac{(1-x^3)(0)-(-7)(-3x^2)}{(1-x^3)^2}=\frac{-21x^2}{(1-x^3)^2}$$

18. $y=\frac{4}{3}x^{\left(\frac{3}{4}-\pi\right)}$

$$y'=\left(\frac{3}{4}-\pi\right)\frac{4}{3}x^{\frac{3}{4}-\pi-1}=\left(\frac{3}{4}-\pi\right)\frac{4}{3}x^{\left(\frac{-1}{4}-\pi\right)}$$

rewrite 19. $y=\frac{1}{7x}=\frac{1}{7}x^{-1}$

$$y'=-\frac{1}{7}x^{-2}$$

20. $y=2x^{\left(\frac{1}{2}-e\right)}$

$$y'=\left(\frac{1}{2}-e\right)2x^{\left(\frac{1}{2}-e-1\right)}=\left(\frac{1}{2}-e\right)2x^{\left(-\frac{1}{2}-e\right)}$$

Bonus:

rewrite $y=e^{\ln x^2}-3x^{-7}$

$$y=x^2-3x^{-7}$$

$$y'=2x+21x^{-8}$$