MATH 108 Quiz #1 Fall 2017 Name Solutions

Directions: Answer each question as completely as possible. Show all work for credit. Good luck.

1. Demonstrate that the decimal $3.6\overline{12}$ is rational by rewriting it as an integer divided by another integer.

let $x = 3.61212\ldots$
$10x = 36.1212\ldots$
$100x = 361.21212\ldots$

$100x = 361.2121\ldots$
$-\ x \quad - 3.6121\ldots$
$99x = 357.6000$

$x = \frac{357.6}{99} = \boxed{\frac{3576}{990}}$

2. Simplify each of the exponential expressions as completely as possible. Write all answers such that there are no negative exponents.

a) $\left(\frac{-2a^4b}{a^{-3}b^2}\right)^3 = \frac{(-2)^3a^{12}b^3}{a^{-9}b^6} = \frac{-8a^{12}a^9b^3}{b^6} = \boxed{\frac{-8a^{21}}{b^3}}$

b) $(2x^{-4}y^3)^3(3x^2y^{-2})^{-2} = 2^3x^{-12}y^9\,3^{-2}x^{-4}y^4 = 8\cdot\frac{1}{9}\cdot x^{-16}\cdot y^{13}$

$= \boxed{\frac{8y^{13}}{9x^{16}}}$

3. Simplify each of the radical expressions as completely as possible. Combine like terms where possible.

a) $\sqrt[3]{72a^4b^5} = \sqrt[3]{8\cdot 9\cdot a^3\cdot a\cdot b^3\cdot b^2}$

$= \boxed{2ab\sqrt[3]{9ab^2}}$

b) $9a\sqrt{20a^3b^2} + 7b\sqrt{45a^5}$

$= 9a\sqrt{4\cdot 5\cdot a^2\cdot a\cdot b^2} + 7b\sqrt{9\cdot 5\cdot a^4\cdot a}$

$= 9a\cdot 2\cdot a\cdot b\sqrt{5a} + 7b\cdot 3\cdot a^2\sqrt{5a}$

$= 18a^2b\sqrt{5a} + 21a^2b\sqrt{5a} = \boxed{39a^2b\sqrt{5a}}$

MATH 108 Quiz #2 Fall 2017 Name Solutions

Directions: Answer each question as completely as possible. Show all work for credit. Good luck.

1. Multiply each of the following expressions. Combine like terms where possible.

a) $(2x^2 + 3x - 5)(x^2 - 7x + 2)$ $= 2x^4 - 14x^3 + 4x^2$
$+ 3x^3 - 21x^2 + 6x$
$- 5x^2 + 35x - 10$

$\boxed{2x^4 - 11x^3 - 22x^2 + 41x - 10}$

b) $(\sqrt{x} - y)(\sqrt{x} + y)$ $= \sqrt{x}\sqrt{x} + \cancel{y\sqrt{x}} - \cancel{y\sqrt{x}} - y^2$

$= \boxed{x - y^2}$

2. Factor each of the following expressions completely.

CF a) $12x^3y - 22x^2y + 8xy$ $= 2xy(6x^2 - 11x + 4)$
$= 2xy(6x^2 - 8x - 3x + 4)$
$= 2xy[(6x^2 - 8x) - 1(3x - 4)]$

$2xy[2x(3x-4) - 1(3x-4)]$

$= \boxed{2xy[3x-4][2x-1]}$

~~FBG~~ FBG b) $(4x^3 + 12x^2) - (9x + 27)$ $= 4x^2(x+3) - 9(x+3)$
$= (x+3)(4x^2 - 9)$
$= \boxed{(x+3)(2x-3)(2x+3)}$

SOC c) $125x^6 + 27y^3$

$= \boxed{(5x^2 + 3y)(25x^4 - 15x^2y + 9y^2)}$

3. Combine the rational expressions. Write your answer in simplest form. Be sure to specify any restrictions on the value of the variable.

$$\frac{7}{x-1}+\frac{10x}{x^2-1}-\frac{5}{x+1} = \frac{7(x+1)+10x-5(x-1)}{x^2-1}$$

$$= \frac{7x+7+10x-5x+5}{x^2-1} = \boxed{\frac{2x+12}{x^2-1}}$$

4. Solve each of the following equations for x.

a) $x^5 - 25x^3 = 0$

$$x^3(x^2-25)=0$$

$$x^3=0 \rightarrow \boxed{x=0} \qquad x^2=25 \rightarrow \boxed{x=\pm 5}$$

b) $\frac{x^2-20}{x^2-7x+12} = \frac{3}{x-3}+\frac{5}{x-4}$

$$\frac{x^2-20}{\cancel{(x-4)(x-3)}} = \frac{3(x-4)}{\cancel{(x-3)(x-4)}} + \frac{5(x-3)}{\cancel{(x-4)(x-3)}}$$

$$x^2-20 = 3x-12+5x-15$$

$$x^2-8x-5=0$$

$$x = \frac{8 \pm \sqrt{64-4(1)(-5)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{84}}{2}$$

$$= \frac{8 \pm 2\sqrt{21}}{2} = \boxed{4 \pm \sqrt{21}}$$

c) $\sqrt{2x+4} = \sqrt{x+3}+1$

$$2x+4 = x+3+2\sqrt{x+3}+1$$

$$2x-x+4-3-1 = 2\sqrt{x+3}$$

$$x = 2\sqrt{x+3}$$

$$x^2 = 4(x+3)$$

$$x^2-4x-12=0$$

$$(x-6)(x+2)=0$$

$x=6, -2$ – Check? Only $\boxed{x=6 \text{ works}}$

Quiz #3 Solutions

1. Solve each of the following quadratic equations for x.

a) $2x^2 - 9x = -9$

$2x^2 - 9x + 9 = 0$

$2 \cdot 9 = 18 = 6 \cdot 3$ or $(-6)(-3)$

$-6 + -3 = -9$

$(2x^2 - 6x) - (3x - 9) = 0$

$2x(x-3) - 3(x-3) = 0 \longrightarrow (2x-3)(x-3) = 0$

$\boxed{x = \frac{3}{2}, 3}$

b) $(3x - 7)^2 = 71$

$3x - 7 = \pm\sqrt{71}$

$3x = 7 \pm \sqrt{71}$

$\boxed{x = \frac{7 \pm \sqrt{71}}{3}}$

c) $x^2 - 2x = 4$

$x^2 - 2x - 4 = 0$

$x = \frac{+2 \pm \sqrt{4 - 4(1)(-4)}}{2(1)} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2}$

$= \boxed{1 \pm \sqrt{5}}$

d) $4x^2 + 7x - 9 = 0$

$x = \frac{-7 \pm \sqrt{49 - 4(4)(-9)}}{2(4)}$

$36 \cdot 4$
144

$= \frac{-7 \pm \sqrt{49 + 144}}{8} = \boxed{\frac{-7 \pm \sqrt{193}}{8}}$

Quiz #4 Take-home Key

1. a) D_f where $f(x) = \dfrac{x^2 - 9x - 36}{2x^2 + 7x + 3}$

$2x^2 + 7x + 3 \neq 0$

$(2x+1)(x+3) \neq 0 \rightarrow \boxed{x \neq -1/2, -3}$

$\rightarrow (-\infty, -3) \cup (-3, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

b) D_g where $g(x) = \sqrt[3]{x-5}$

$x - 5$ may be $>$, $<$ or $=$ zero

so $x = \mathbb{R} \rightarrow (-\infty, \infty)$

c) $h(x) = \begin{cases} x^2, & x \text{ even integer} \\ x-1, & x \text{ odd integer} \end{cases}$

$D_h: x = \mathbb{Z}$ or $\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

Quiz 4

Domain

1. c) $h(x) = \begin{cases} x^2, & x \text{ even integer} \\ x-1, & x \text{ odd integer} \end{cases}$

Dom: $x \in \mathbb{Z}$ or $\{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$

d) $f(x) = \begin{cases} 6, & x \le 2 \\ x+1, & x > 5 \end{cases}$ Dom $(-\infty, 2] \cup (5, \infty)$

2. $f(x) = \dfrac{1}{x^4 - 9}$, $g(x) = \sqrt{x-1}$

a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-1})$

↳ Look at input domain first: $x - 1 \geq 0$

so $x \geq 1$

$$= \frac{1}{\underbrace{(\sqrt{x-1})^4 - 9}_{\neq 0}}$$

ready to simplify

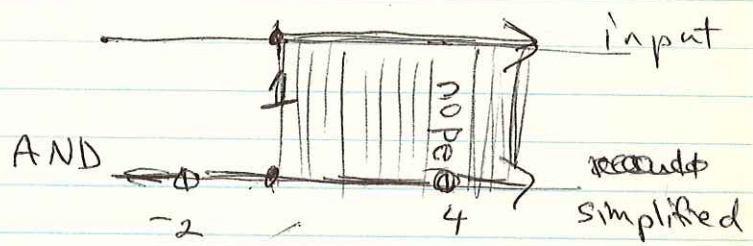
$((x-1)^{1/2})^4 - 9$

$= (x-1)^2 - 9$

$= x^2 - 2x + 1 - 9 = x^2 - 2x - 8$

$= (x-4)(x+2) \neq 0$

$x \neq 4, -2$



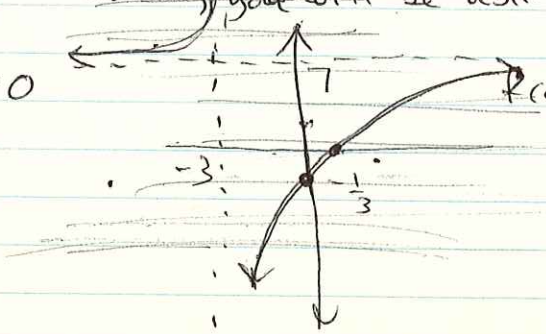
$[1, 4) \cup (4, \infty)$

3. $f(x) = \dfrac{7x-1}{x+3}$

Later, you will be able to graph this:

$f(x) = 0$ where $7x - 1 = 0$

$x = \frac{1}{7}$



$f(0) = \dfrac{-1}{3}$

1.c) $h(x) = \begin{cases} x^2, & x \text{ even integer} \\ x-1, & x \text{ odd integer} \end{cases}$

Dom $x \in \mathbb{Z}$ or $\{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$

d) $f(x) = \begin{cases} 5, & x \le 5 \\ x+1, & x > 5 \end{cases}$ Dom $(-\infty, 5] \cup (5, \infty)$

a) $f(x) = \dfrac{1}{x^4 - 1}$; $g(x) = \sqrt{x-1}$

a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-1})$

↳ Look at $g(x)$ first

$x - 1 \ge 0$

$1 \le x$

$\dfrac{1}{(\sqrt{x-1})^4 - 1}$

$\neq 0$

$(\sqrt{x-1})^4 - 1 \neq 0$

$(x-1)^2 - 1 \neq 0$

$(x-1)^2 - 1$

$x^2 - 2x + 1 - 1 = x^2 - 2x - 3$

$(x-1)(x+3) \neq 0$

$x \neq 1, -3$

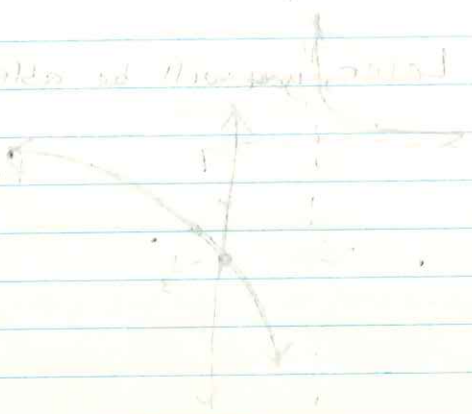
Dom

$[1, 1) \cup (1, \infty)$

3. $f(x) = \dfrac{x-1}{|x|+3}$

$f(x)$ is when $|x-1| = 0$

$x = 1$



a) Use def of 1-1, show $f(x)$ is indeed 1-1.

① Suppose $f(a) = f(b)$

That is $\frac{7a-1}{a+3} = \frac{7b-1}{b+3}$

② Conclude by simplifying (algebra) that $a=b$

$$(7a-1)(b+3) = (a+3)(7b-1)$$

$$\cancel{7ab} + 21a - b - \cancel{3} = \cancel{7ab} - a + 21b - \cancel{3}$$

$$21a + a = 21b + b$$

$$\frac{\cancel{22}a}{\cancel{22}} = \frac{\cancel{22}b}{\cancel{22}} \rightarrow a = b \checkmark$$

f is 1-1

b) $f(x) = \frac{7x-1}{x+3}$ Find $f^{-1}(x)$ (it exists f is 1-1)

① $y = \frac{7x-1}{x+3}$

② $x = \frac{7y-1}{y+3}$

③ Isolate y with algebra

$$x(y+3) = 7y - 1$$

$$xy + 3x = 7y - 1$$

$$3x + 1 = 7y - xy$$

$$\frac{3x+1}{7-x} = \frac{y\cancel{(7-x)}}{\cancel{(7-x)}}$$

$$y = \frac{3x+1}{7-x}$$

④ $f^{-1}(x) = y$

c) Find the range of f. ∴ Wait: $\text{Dom } f^{-1} = \text{Ran } f$

$\text{Dom } f^{-1}$: $x \neq 7$ $\text{Ran } f$: $y \neq 7$

d) Show $f(x)$ is neither odd nor even:

① $f(-x) = \dfrac{7(-x)-1}{-x+3} = \dfrac{-7x-1}{-x+3} \overset{?}{=} f(x)$

$$\frac{7x(-1)}{x+3} \neq \frac{-7x(-1)}{-x+3} = \frac{-(7x+1)}{-x+3}$$

② Compare $f(-x)$ to $-f(x) = -\left(\dfrac{7x-1}{x+3}\right)$

$$\frac{-7x-1}{-x+3} \overset{?}{\neq} \frac{-7x+1}{x+3}$$

So $f(x)$ is neither.

d) Show $f(x)$ is neither odd nor even:

① $f(-x) = \frac{1-(-x)^2}{-x+3} = \frac{1-x^2}{-x+3} \neq f(x)$

$\frac{(1-x^2)}{-x+3} = \frac{-(x^2-1)}{-x+3} = \frac{x^2-1}{x+3}$

② Compare $f(-x)$ to $-f(x) = \left(\frac{1-x^2}{x+3}\right)$

$\frac{1-x^2}{-x+3} \neq \frac{x^2-1}{x+3}$

$\therefore f(x)$ is neither

Solving Quadratics

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

$(x-3)^2 = 1$

$\sqrt{(x-3)^2} = \pm\sqrt{1}$

$x - 3 = \pm 1$

$x = 3 \pm 1$

$x = 4, 2$

$x^2 - 5x = 8$

$x^2 - 5x - 8 = 0$

CTS

QF

$x = \dfrac{5 \pm \sqrt{25+32}}{2}$

$= \dfrac{5 \pm \sqrt{57}}{2}$

$x^2 - 5x + \frac{25}{4} = 8 + \frac{25}{4}$ $\qquad \left(\frac{b}{2}\right)^2$

$\left(x - \frac{5}{2}\right)^2 = \frac{57}{4}$

$x - \frac{5}{2} = \pm\sqrt{\frac{57}{4}}$

$x = \frac{5}{2} \pm \sqrt{\frac{57}{4}} = \dfrac{5 \pm \sqrt{57}}{2}$

Ex of CTS $\qquad 3x^2 - 6x + 11 = 0$

$3x^2 - 6x = -11$

$3(x^2 - 2x + 1) = -11 + 3$

$3(x-1)^2 = -8$

$\sqrt{(x-1)^2} = \sqrt{-\frac{8}{3}}$

no soln
(real)