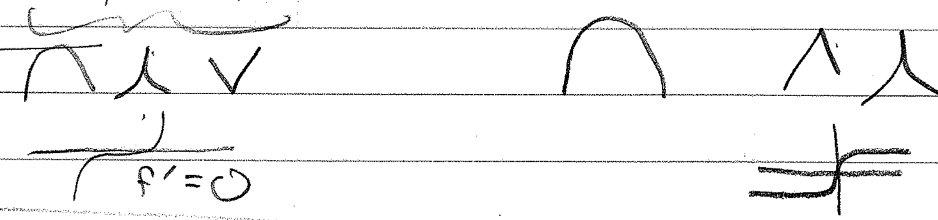
Def "Critical point" - a point at $x=a$ in the domain of $f(x)$ where either

$f'(a)=0$ or DNE

i.e. max, min, $\qquad f'(a)=0 \qquad f(a)$ DNE



Review - higher order derivative exs.

Ex 1

$$f(x) = 2e^{(3x-1)}$$
$$f'(x) = 2(e^{3x-1}\cdot 3) = 6e^{3x-1}$$
$$f''(x) = 6(e^{3x-1}\cdot 3) = 18e^{3x-1}$$

Ex 2

$$f(x) = x^5 - 4x^3 + 2x - 7$$
$$f'(x) = 5x^4 - 12x^2 + 2$$
$$f''(x) = 20x^3 - 24x$$
$$f'''(x) = 60x^2 - 24$$
$$f^{(4)}(x) = 120x$$
$$f^{(5)}(x) = 120$$
$$f^{(6)}(x) = 0$$

__Ex__ $f(x) = (x^2-x)^{1/3}$

$f'(x) = \frac{1}{3}(x^2-x)^{-2/3}(2x-1)$

__Aside__ When asked to solve

$f(x) = \frac{P(x)}{Q(x)} = 0$

you need only find x where $P(x)=0$.

$f(x) = \frac{2x-1}{3(x^2-x)^{2/3}} = 0$

So $2x-1=0$ at $\boxed{x=\frac{1}{2}}$

When asked for critical values, you must then also check where $f'(x)$ DNE.

__Isolate__ denominator

$(x^2-x)^{2/3} \stackrel{?}{=} 0$

$(x(x-1))^{2/3} = 0$

Clear the exponent?

$\left((x(x-1))^{\cancel{2/3}}\right)^{\cancel{3/2}} = 0^{3/2}$

$x(x-1) = 0$

$\boxed{x=0,1}$ $f'(x)$ DNE

HW Look for $f'(x) = 0$ or DNE

#4 $f(x) = x^2\sqrt{x-4} = x^2(x-4)^{1/2}$

$f'(x)$ by product rule:

$$f'(x) = 2x\,(x-4)^{1/2} + x^2 \cdot \frac{1}{2}(x-4)^{-1/2}$$

$$f'(x) \stackrel{?}{=} 0$$

First change the negative exponents

$$0 = \frac{2x(x-4)^{1/2}}{1} + \frac{x^2}{2(x-4)^{1/2}}$$

Cannot see the solution until $\frac{P(x)}{Q(x)}$ form

Needs an LCD: $2(x-4)^{1/2}$

$$0 = \frac{2(x-4)^{1/2}}{2(x-4)^{1/2}} \cdot \frac{2x(x-4)^{1/2}}{1} + \frac{x^2}{2(x-4)^{1/2}}$$

$$= \frac{4x(x-4) + x^2}{2(x-4)^{1/2}}$$

$$= \frac{4x^2 - 16x + x^2}{2(x-4)^{1/2}} = \frac{5x^2 - 16x}{2(x-4)^{1/2}}$$

$$= \frac{x(5x-16)}{2(x-4)^{1/2}} = 0$$

$f' = 0$

① Look at $x(5x-16) = 0 \rightarrow \boxed{x = 0, \frac{16}{5}}$

② Look at $2(x-4)^{1/2} = 0 \rightarrow \boxed{x = 4}$ DNE

(3) Check that the values found in (1) + (2) are in the domain of the original fcn.

$$f(x) = x^2\sqrt{x-4}$$

Dom: $x - 4 \geq 0$

$x \geq 4$

So discard $x = 0 + \frac{16}{5}$