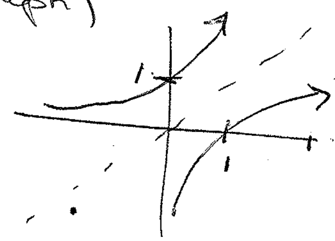
Math 108 Fall 2014 Quiz #8 Name Key

Directions: Answer each question as completely as possible. Show all work for credit. Good luck.

1. Graph $f(x) = 3^{x-1} + 1$. Give the coordinates of the y-intercept and the equation of the asymptote.

$f(0) = 3^{0-1} + 1 = \frac{1}{3} + 1 = 1\frac{1}{3}$; HA: $y = 1$

2. Graph $f(x) = e^x$ and $g(x) = \ln(x)$ on the same set of axes. (see graph)



$f(x) = g^{-1}(x) \longrightarrow f(0) = 1, \quad g(1) = 0$

$f^{-1}(x) = g(x) \qquad f(1) = e, \quad g(e) = 1$

3. Evaluate each of the following:

a) $\log_2 16 = y \longrightarrow \log_2 2^4 = 4\log_2 2 = 4 \cdot 1 = \boxed{4}$

b) $\log_{\frac{1}{2}} 8 = y \rightarrow \log_{\frac{1}{2}} 2^3 = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-3} = -3\log_{\frac{1}{2}}\left(\frac{1}{2}\right) = -3 \cdot 1 = \boxed{-3}$

c) $\ln 1 = \boxed{0}$

d) $3^{\log_3 18} = \boxed{18}$

e) $\log_3 72 - \log_3 8 = \log_3\left(\frac{72}{8}\right) = \log_3 9 = \boxed{2}$

4. Change to an equivalent expression using natural logarithms: $\log_6 12$

Prop $\log_b a = \frac{\log_c a}{\log_c b} \longrightarrow$ letting $c = e$ $\quad \boxed{\frac{\ln 12}{\ln 6}}$

5. Solve each equation for x.

a) $4^x = 2^{3x-5}$

b) $\ln(x + 1) = -1$

c) $\log_3 x - \log_3 5 = 2$

d) $\log_5 x + \log_5(x - 2) = \log_5(6 - x)$

e) $5^x = 7^{x+2}$

d) $\log_5[x(x-2)] = \log_5(6-x)$

$x(x-2) = 6 - x$

$x^2 - 2x + x - 6 = 0$

$x^2 - x - 6 = 0 \rightarrow (x-3)(x+2) = 0$

$\boxed{x = 3, -2}$ discard

check

b) $e^{-1} = x + 1 \rightarrow \frac{1}{e} = x + 1$

$x = \frac{1}{e} - 1$ or $\frac{1-e}{e}$

a) $(2^2)^x = 2^{3x-5}$

$2^{2x} = 2^{3x-5}$ $\quad \boxed{x = 5}$

$2x = 3x - 5$

c) $\log_3\left(\frac{x}{5}\right) = 2 \rightarrow \frac{x}{5} = 3^2$

$x = 9 \cdot 5 = \boxed{45}$

6. Solve each system of equations.

a) $\begin{cases} 5x - 2y = 12 \\ -15x + 6y = 21 \end{cases}$ $\longrightarrow$ $\begin{aligned} \cancel{15x} - \cancel{6y} &= 36 \\ -\cancel{15x} + \cancel{6y} &= 21 \\ \hline 0 &\neq 57 \end{aligned}$ $\rightarrow$ no soln. parallel lines

b) $\begin{cases} x = y^2 \\ y = -x + 6 \end{cases}$ $\longrightarrow$ $\begin{aligned} y^2 &= \cancel{x} \\ y &= \cancel{-x} + 6 \\ \hline y^2 + y &= 6 \end{aligned}$ $\rightarrow$ $y^2 + y - 6 = 0$

$(y-2)(y+3) = 0$ $\rightarrow$ $\boxed{\begin{aligned} y &= 2 \\ x &= 4 \end{aligned}}$ $\quad$ $\boxed{\begin{aligned} y &= -3 \\ x &= 9 \end{aligned}}$

7. Solve the system of inequalities. Find coordinates of all vertices.

$\begin{cases} y \geq 2x^2 - 3 \\ y < -2x + 1 \end{cases}$ Graph is seen below.

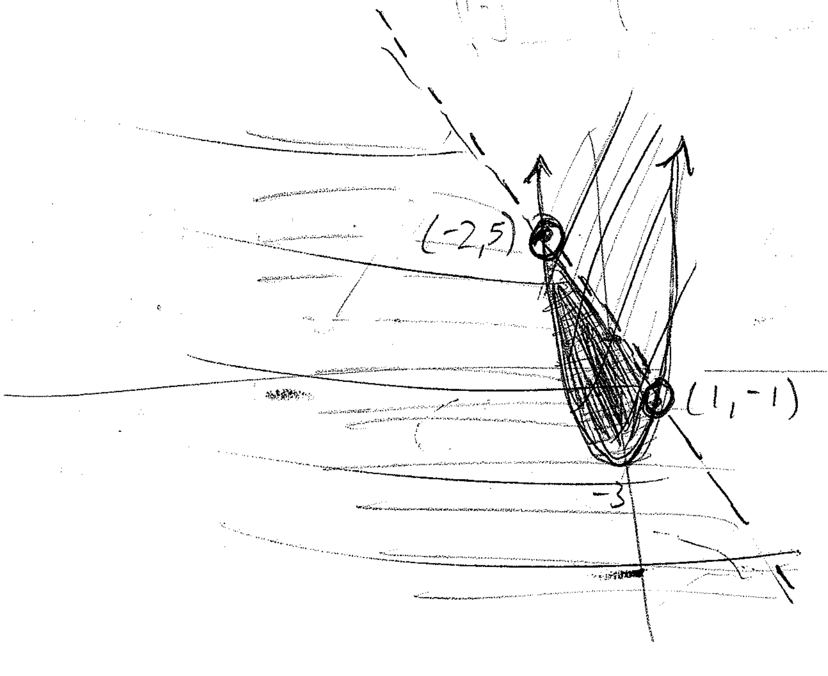
$\Rightarrow$ Set $2x^2 - 3 = -2x + 1$ to find vertices $\Leftarrow$

$2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$ $\rightarrow$ $\boxed{\begin{aligned} x &= 1 \\ y &= -1 \\ &(1,-1) \end{aligned}}$ $\quad$ $\boxed{\begin{aligned} x &= -2 \\ y &= 5 \\ &(-2,5) \end{aligned}}$

The vertices are not part of the soln. set.



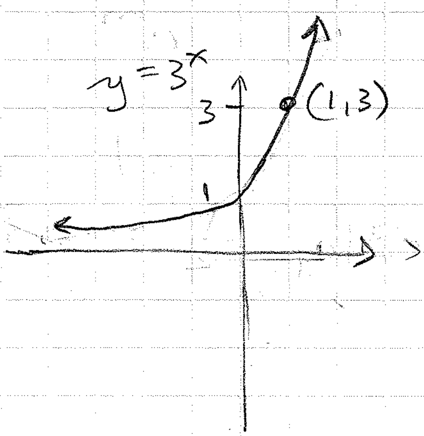
Quiz 8 Fall 2014 Key

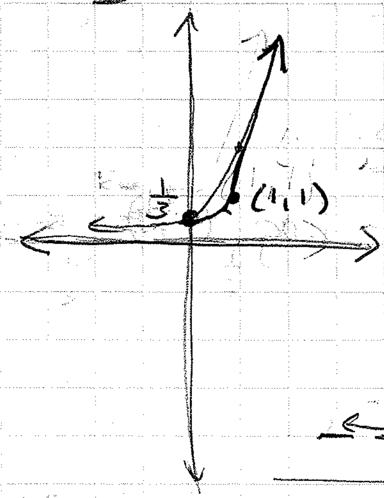
#1 This is a transformation of $y = 3^x$, where we recognize 3^{x-1} as $\frac{3^x}{3^1}$

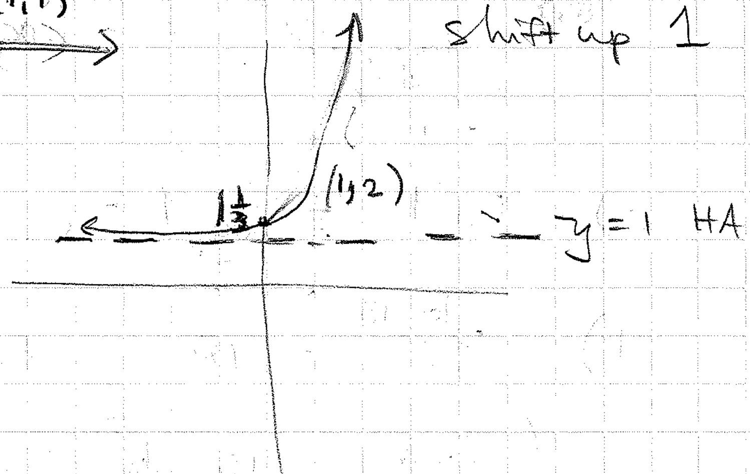
$$y = 3^{x-1} + 1 = 3^x 3^{-1} + 1 = \frac{1}{3}\cdot 3^x + 1$$

$$= \frac{3^x}{3} + 1$$

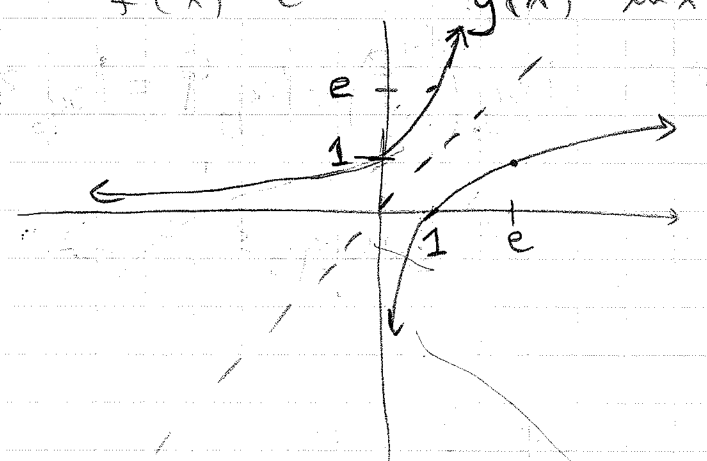
$$= \underbrace{\underbrace{\frac{1}{3}\underbrace{\left(3^x\right)}_{\text{mother fcn}}}_{\text{compression by } \frac{1}{3}} + 1}_{\text{shift up } 1}$$







#2 $f(x) = e^x$ $g(x) = \ln x$



I've shown the y-int = 1 and f(1) = e for f(x), and that the x-int = 1 for $g(x) = f^{-1}(x)$ (also, g(e) = 1)

3a) $\log_2 16 = y \xleftrightarrow{def} 16 = 2^y \rightarrow y = 4$

OR

$\cancel{\log_2 2^4}$

$\log_2 2^4 = 4\cancel{\log_2 2}^{1} = 4 \cdot 1 = 4$

b) $\log_{\frac{1}{2}} 8 = y \xleftrightarrow{def} 8 = \frac{1}{2}^y \rightarrow y = -3$

check: $\left(\frac{1}{2}\right)^{-3} = 2^3 = 8 \qquad a^{-m} = \frac{1}{a^m}$

OR

$\log_{\frac{1}{2}} 2^3 = \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-3} = -3\cancel{\log_{\frac{1}{2}}\frac{1}{2}}^{1} = -3$

c) $\ln 1 \equiv \log_e 1 = 0$

d) $3^{\log_3 18} = 18 \quad$ b/c $\quad b^{\log_b x} = x$

(Twin: $\log_b b^x = x$)

e) $\log_3 72 - \log_3 8 = \log_3\left(\frac{72}{8}\right) = \log_3 9 = \log_3 3^2$

$= 2\cancel{\log_3 3}^{1} = 2$

~~4~~ 5) a) check $x = 5$: $4^5 \stackrel{?}{=} 2^{10}$

$(2^2)^5 = 2^{10}$ ✓

b) $e^{-1} = x + 1$

$\boxed{x = e^{-1} - 1}$ ok

5.e) $5^x = 7^{x+2}$

$$\log_7 5^x = \log_7 7^{x+2}$$

$$x \log_7 5 = (x+2)\cancel{\log_7 7}^{\,1}$$

$$x\log_7 5 = x+2$$

$$x\log_7 5 - x = 2$$

factor x out

$$x(\log_7 5 - 1) = 2$$

$$\boxed{x = \frac{2}{(\log_7 5) - 1}}$$

$$a^{f(x)} = b^{g(x)}$$

$$\log_a a^{f(x)} = \log_a b^{g(x)}$$

$$f(x) = g(x)\boxed{\log_a b}$$

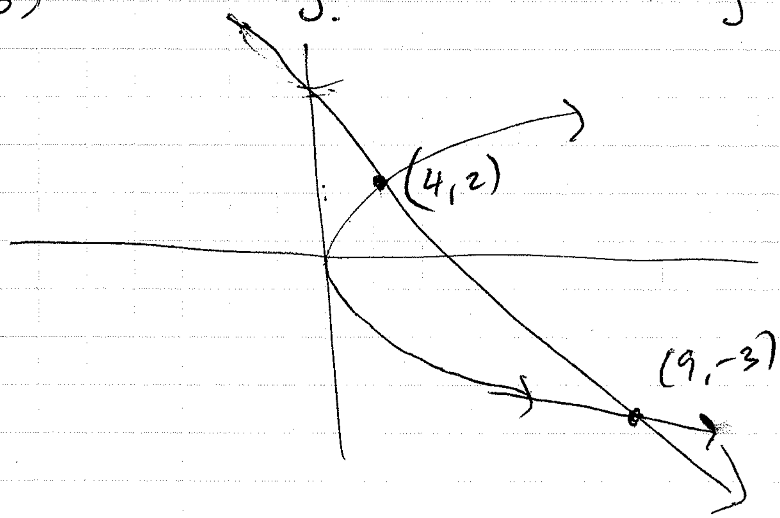
just a ~~const.~~ coeff.

$\downarrow$

Then solve for x in terms of the horrible looking log number.

- bring x terms together

b) $x = y^2$ $\qquad y = -x + 6$



You didn't have to graph this, but I'm showing soln. drawing. Scale is poor.