Solutions to Sec 1.6

(a) $x^3 - 4x = 0$

$x(x^2-4) = 0$

$x(x+2)(x-2) = 0$

$x = 0,\ x+2 = 0\ \ x-2=0$, so $\boxed{x = 0, -2, 2}$

(b) $x^3 + 4x = 0$

$x(x^2+4) = 0$

$x = 0\ \ x^2+4 = 0$, so $\boxed{x = 0 \text{ only}}$

because ~~setting~~ we cannot solve

$x^2+4 = 0$ in the set of real numbers.

Why not? Setting $x^2+4 = 0$

we could try to factor, but we already know the sum of squares does not factor over the reals

In other words, there are no real numbers r_1, r_2 such that $x^2+4 = (x-r_1)(x-r_2)$

If there were, then $x^2 = -4$ would have a soln. in the reals, but no real number gives $x^2 = -4$.

Hence, $x = 0$ is the only soln.

(c) $-2x^2 - 15x + 27 = 0$

Factor out the negative leading coeff.

$-(2x^2 + 15x - 27) = 0$

Unfortunately, the rest is trial + error by reverse FOIL. (Though later we'll see how to factor using the quadratic formula.)

$$0 = -(2x^2 + 15x - 27) = -(2x - 3)(x + 9)$$

Since $-(2x-3)(x+9) = 0$

then $2x - 3 = 0$ or $x + 9 = 0$

Thus: $\boxed{x = 3/2 \quad \text{or} \quad x = -9}$

(d) $x^3 - 2x^2 = 3x$ Bring all terms to one side, setting = zero.

$x^3 - 2x^2 - 3x = 0$

$x(x^2 - 2x - 3) = 0$ Factor out x.

$x(x-3)(x+1) = 0$ Factor further

So $\boxed{x = 0, 3, \text{ or } -1}$

(e) $x(x+2) = 99$ You can't solve it as is. You need to use the zero property of X, ~~so~~ but you can't when it's not set = zero.

$x(x+2) - 99 = 0$ Expand.

$x^2 + 2x - 99 = 0$ Factor.

$(x-9)(x+11) = 0$ Right?

$$\boxed{x = 9 \text{ or } -11}$$

(f) $$\frac{x^2 - 4x}{x+3} = \frac{5}{x+3}$$

Before you 'cross multiply' (a misnomer, but never mind yet), notice that the denominators are equal, so you need only solve the problem:

$$x^2 - 4x = 5$$

but keep in mind that in your final answer, $x \neq -3$, since in the original statement, the denominator $x+3 = 0$ at $x = -3$. So we're restricted to solns where $x \neq -3$.

$$x^2 - 4x = 5 \longrightarrow x^2 - 4x - 5 = 0$$

$$\longrightarrow (x-5)(x+1) = 0 \longrightarrow \boxed{x = 5 \text{ or } -1} \, //$$

(g) $x(3x-23) = 8$ Again, set = zero.

$x(3x-23) - 8 = 0$ Expand

$3x^2 - 23x - 8 = 0$ Factor by reverse FOIL (trial + error ¨)

$(3x + 1)(x - 8) = 0$

So $3x+1 = 0$ or $x - 8 = 0$

hence $\boxed{x = -1/3 \text{ or } 8}$

(h) $\frac{x}{4}(x+1) = 3$

Clear the denom.

$x(x+1) = 12$

$x(x+1) - 12 = 0$ Set = zero

$x^2 + x - 12 = 0$ Expand

$(x-3)(x+4) = 0$ Factor + solve

$\boxed{x = 3 \text{ or } -4}$

(i) $\frac{3}{x+5} + \frac{4}{x} = 2$

Find the common denominator - since $x+5$, x, + 1 have no factors in common, the LCD will be their product $x(x+5)$.

$$\frac{3}{x+5} \cdot \frac{x}{x} + \frac{4}{x} \cdot \frac{x+5}{x+5} = 2 \cdot \frac{x+5}{x+5} \cdot \frac{x}{x}$$

$3x + 4(x+5) = 2(x^2+5x)$ Since all terms now have a CD, you may eliminate it to solve

$3x + 4x + 20 - 2x^2 - 10x = 0$

~~$5x + 10 = 0$~~

$-2x^2 - 3x + 20 = 0$ Divide all by -1

$2x^2 + 3x - 20 = 0$ Factor + solve

$(2x-5)(x+5) = 0$

$\boxed{x = 5/2,\ -5}$ — Check these into the original + make sure both are allowed.

Both solns. check out + we note that the restriction on the original

$3/x+5 + 4/x = 2$

is that $x \neq -5$ or 0.

(j) $\sqrt{x+7} = x - 13$ Square both sides to eliminate the radical.

$x+7 = (x-13)^2$ Expand.

$x+7 = x^2 - 26x + 169$

$x^2 - 27x + 162 = 0$ Set = zero + solve.

$(x-18)(x-9) = 0$

$\boxed{x = 18, 9}$ possible solns. Check it!

① $\sqrt{18+7} \stackrel{?}{=} 18-13$

$\sqrt{25} = 5 = 18-13$ ✓

② $\sqrt{9+7} \stackrel{?}{=} 9-13$

$\sqrt{16} = 4 \neq -4$

Since $x = 9$ gives a radical = negative, that soln. is discarded.

$\boxed{x = 18}$ Only soln.

(k) $\sqrt{5x+9} - x = -1$

Bring nonradical terms to one side. Square each side.

$\sqrt{5x+9} = x - 1$

$5x+9 = (x-1)^2$

$5x + 9 = x^2 - 2x + 1$

Set = zero + solve.

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$ so $\boxed{x = 8, -1 \text{ possible solns}}$

Check

① $\sqrt{5 \cdot 8 + 9} - 8 \stackrel{?}{=} -1$

$\sqrt{49} - 8 = 7 - 8 = -1$ ✓ ok

② $\sqrt{5(-1)+9} + 1 \stackrel{?}{=} -1$

$\sqrt{4} + 1 = 2 + 1 \neq -1$ discard $x = -1$

$\boxed{\text{Soln } x = 8}$

(l) $\sqrt{3x-2} = 2 + \sqrt{x}$

Since bringing the radicals together will result in a radical after squaring, this is not a good approach. If we square both sides first, we'll still have a radical, but it will be simpler.

$$\sqrt{3x-2} = 2 + \sqrt{x}$$ Square both sides.

$$3x-2 = (2+\sqrt{x})^2$$ Expand.

$$3x-2 = 4 + 4\sqrt{x} + x$$

$$2x-6 = -4\sqrt{x}$$ Now isolate the radical.

or

$$x-3 = -2\sqrt{x}$$ Square both sides again.

$$(x-3)^2 = 4x$$ Expand. Set equal to zero.

$$x^2 - 6x + 9 = 4x$$ Solve.

$$x^2 - 10x + 9 = 0$$

$$(x-9)(x-1) = 0$$ $x = 9, 1$ possible solns

Check both + reject $x = 1$ (Why?)

(m) $\sqrt{3x+6} - \sqrt{x+4} = \sqrt{2}$

We just saw that squaring both sides with radicals eventually eliminates them. We'll do this in class and see what happens!