Math 220 - Calculus f. Business and Management - Worksheet 15

Solutions for Worksheet 15 - e^x and the Chain Rule

Exercise 1: *Find the derivative of each function*

$$\textbf{1a}: f(x) = e^x\, x^2, \qquad \textbf{1b}: f(x) = (x^3 - 2x^2 + 5)\,(e^x + x - 2),$$
$$\textbf{1c}: f(x) = \frac{e^x}{2x^2 + 3x - 7}, \qquad \textbf{1d}: f(x) = \frac{\sqrt{x}}{5e^x}.$$

Solution to #1:

For 1a: Product rule. $\frac{d}{dx}e^x x^2 = e^x x^2 + e^x 2x.$

For 1b: Product rule again. $\frac{d}{dx}(x^3 - 2x^2 + 5)\,(e^x + x - 2) = (3x^2 - 4x)(e^x + x - 2) + (x^3 - 2x^2 + 5)(e^x + 1).$

For 1c: Quotient rule. $\frac{d}{dx}\Big(\frac{e^x}{2x^2 + 3x - 7}\Big) = \frac{e^x(2x^2 + 3x - 7) - e^x(4x + 3)}{(2x^2 + 3x - 7)^2} = \frac{e^x(2x^2 - x - 10)}{(2x^2 + 3x - 7)^2}.$

For 1d: Simplify, then use quotient rule. $\frac{d}{dx}\Big(\frac{\sqrt{x}}{5e^x}\Big) = \frac{d}{dx}\Big(\frac{x^{1/2}}{5e^x}\Big) = \frac{5e^x((1/2)x^{-1/2}) - x^{1/2}(5e^x)}{(5e^x)^2}.$

Exercise 2*: Decompose the functions $f(x)$ into $f(u)$ and $u(x)$*

$$\textbf{2a}: f(x) = (6x^4 + 3x - 8)^5, \qquad \textbf{2b}: f(x) = \sqrt[3]{x^2 - 5x}, \qquad \textbf{2c}: f(x) = \Big(\frac{5}{x} + 7\Big)^3.$$

Solution to #2:

We try to pick the "outer" function to be our $f(u)$.

For 2a: Pick $f(u) = u^5$ and pick $u(x) = (6x^4 + 3x - 8)^5$. Apply the chain rule to get the derivative. So,

$$\frac{d}{dx}(6x^4 + 3x - 8)^5 = 5(6x^4 + 3x - 8)^4 \cdot (24x^3 + 3)$$

.

For 2b: Pick $f(u) = \sqrt[3]{u} = u^{1/3}$. Pick $u(x) = x^2 - 5x$. Now apply the chain rule. So,

$$\frac{d}{dx}\sqrt[3]{x^2 - 5x} = (1/3)(x^2 - 5x)^{-2/3} \cdot (2x - 5)$$

.

For 2c: Pick $f(u) = u^3$ and pick $u(x) = \frac{5}{x} + 7 = 5x^{-1} + 7$. Apply the chain rule. Then,

$$\frac{d}{dx}\Big(\frac{5}{x} + 7\Big)^3 = 3\Big(\frac{5}{x} + 7\Big)^2 \cdot (-5x^{-2}) = (-15x^{-2}) \cdot \Big(\frac{5}{x} + 7\Big)^2$$

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Exercise 3:

$$\textbf{\textit{3a}} : f(x) \;=\; e^{x^2-2x+4}, \qquad \textbf{\textit{3b}} :\; f(x) \;=\; e^{\sqrt{x}}.$$

Solution to #3:

In each problem we simply apply the chain rule. The "inner" function is the contents of the exponent attached to e.

For 3a: Pick $f(u) = e^u$ *and pick* $u(x) = x^2 - 2x + 4$. *Then,*

$$\frac{d}{dx}e^{x^2-2x+4} \;=\; e^{x^2-2x+4} \cdot (2x-2)$$

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For 3b: Pick $f(u) = e^u$ *and pick* $u(x) = \sqrt{x} = x^{1/2}$. *Then,*

$$\frac{d}{dx}e^{\sqrt{x}} \;=\; \frac{d}{dx}e^{x^{1/2}} \;=\; e^{x^{1/2}} \cdot ((1/2)x^{-1/2})$$

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Exercise 4:

$$\textbf{\textit{4a}} : f(x) \;=\; e^{2/x^2+3x-4}, \qquad \textbf{\textit{4b}} :\; f(x) \;=\; \left[e^{(5x^2+2)}\right]^3.$$

Solution to #4:

For 4a: Pick $f(u) = e^u$. *Pick* $u(x) = 2/x^2 + 3x - 4 = 2x^{-2} + 3x - 4$. *Then,*

$$\frac{d}{dx}e^{2/x^2+3x-4} \;=\; \frac{d}{dx}e^{2x^{-2}+3x-4} \;=\; e^{2x^{-2}+3x-4} \cdot (-4x^{-3}+3)$$

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For 4b: First simplify

$$\frac{d}{dx}\left(e^{(5x^2+2)}\right)^3 \;=\; \frac{d}{dx}(e^{15x^2+6}) \qquad (\textit{use} \;\; \left[e^A\right]^B \;=\; e^{A\cdot B}\;)$$

Now, pick $f(u) = e^u$ *and pick* $u(x) = 15x^2 + 6$. *Then*

$$\frac{d}{dx}(e^{15x^2+6}) \;=\; (e^{15x^2+6})(30x)$$

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