

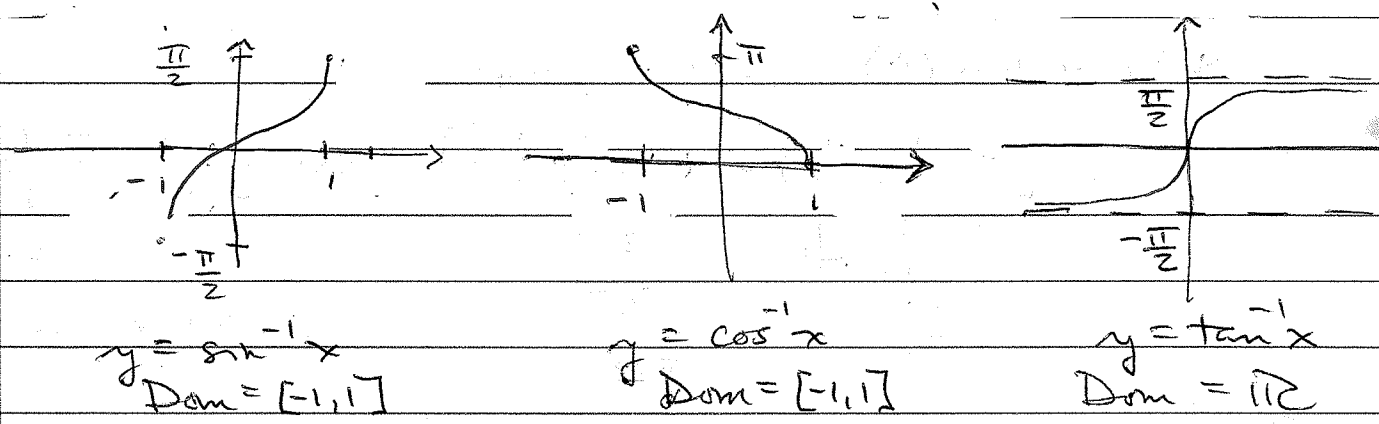
Sec 8.6 Inverse trig fns

Remember, $\arcsin x$, or $\sin^{-1} x$, is defined on a domain of $[-1, 1]$ and since we restricted $\sin x$ to a domain of $[-\pi/2, \pi/2]$ so we could produce the inverse, the range of $\sin^{-1} x$ is also $[-\pi/2, \pi/2]$.

The $\arccos x$, or $\cos^{-1} x$, domain is also $[-1, 1]$ but range reflects choice of $[0, \pi]$ for domain restriction on $\cos x$ that produced the inverse.

#1

Graphs



#2

a) $\text{Dom } f(x) = \cos^{-1}(x+2)$, where $-1 \leq x+2 \leq 1$,
 or $-3 \leq x \leq -1$. Note, too, that the
 fn $y = (\cos x) - 2$ is the inverse of
 $f(x) = \cos^{-1}(x+2)$, and the Range y is $[-3, -1]$.

b) $f(x) = \arctan(x^2 + 3x)$; $x^2 + 3x \in \mathbb{R}$
 so obviously $x \in \mathbb{R}$, that is $\text{Dom } f$ is $(-\infty, \infty)$

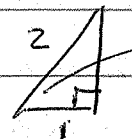
c) $f(x) = \arcsin(1-x)$; $-1 \leq 1-x \leq 1$
 $-2 \leq -x \leq 0$
 $2 \geq x \geq 0$

So $\text{Dom } f$ is $[0, 2]$

#3 a) $\sin^{-1}(-1) = -\frac{\pi}{2}$ (graph)

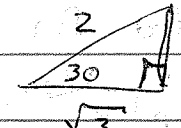
b) $\arcsin(-\sqrt{2}/2) = -\frac{\pi}{4}$ (graph or table value)

c) $\cos^{-1}(0) = \pi/2$ (graph)

d) $\arccos(-\frac{1}{2}) = \frac{2\pi}{3}$ (graph) → Also  60° , but in Q2 since \cos^{-1} is neg.

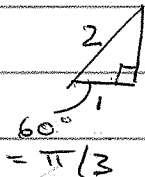
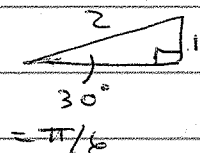
e) $\arcsin(\sqrt{3}/2) = \pi/3$ (table value)

f) $\tan^{-1}(-1) = -\pi/4$ (table value)

g) $\tan^{-1}(1/\sqrt{3}) = \pi/6$ ←  or table

h) $\arctan(0) = 0$ graph

i) $\csc[\sin^{-1}(1/2) - \cos^{-1}(1/2)] = \csc[\frac{\pi}{6} - \frac{\pi}{3}] = \csc(-\frac{\pi}{6})$



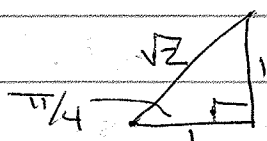
$= -2$

Since $\csc = \frac{1}{\sin}$

#4 This question asks what inverse fun will yield the angle values seen.

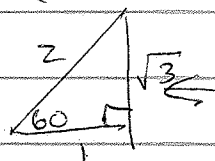
Again, the graph or the trig table is your best source.

(a) $\pi/4 = \arcsin(\sqrt{2}/2) = \arccos(\sqrt{2}/2) = \arctan(1)$



All in Q1 are positive

(b) $-\pi/3$



Look at the graphs, left of zero and from our knowledge of basic trig values (see table), we find

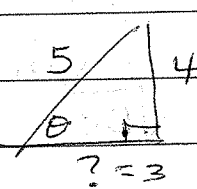
$= \pi/3 = \sin^{-1}(-\sqrt{3}/2) = \cos^{-1}(?)$ No domain value for arccos produces $-\pi/3$
 $= \tan^{-1}(-\sqrt{3})$ → table

Sec 8.6 con'd

#5) a) $\sin^{-1}\left(\frac{4}{5}\right) = \cos^{-1}(?)$

To find $?$, we must construct the right triangle that yielded $\sin^{-1}\left(\frac{4}{5}\right)$, determine the missing side, & express the cosine ratio from this. Notice we won't need to find the angle itself.

$\cos^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{4}{5}\right)$

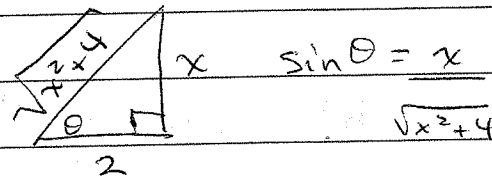


$? = 3$

$$? = \sqrt{25-16}$$
$$= \sqrt{9}$$
$$= 3$$

b) $\tan^{-1}\left(\frac{x}{2}\right) = \sin^{-1}(?)$

$\tan^{-1}\left(\frac{x}{2}\right) = \sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)$



c) $\cos^{-1}(-1) = \pi$, which is not in the range of $\sin^{-1} x$, so $\sin^{-1}(?)$ has no solution.

#4 con'd

e) $\frac{\pi}{2}$ is in range of \sin^{-1} , \cos^{-1} , but not \tan^{-1}

$\frac{\pi}{2} = \sin^{-1}(1) = \cos^{-1}(0)$ are the only two.

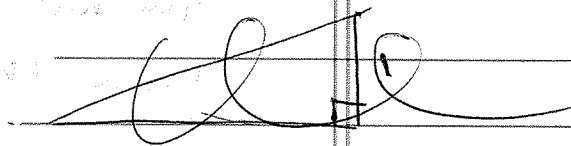
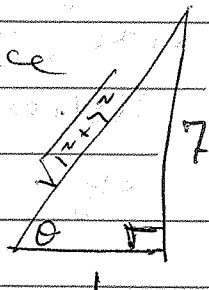
d) 0 is in range of all three:

$0 = \sin^{-1}(0) = \cos^{-1}(1) = \tan^{-1}(0)$

#6) Evaluate w/o a calculator - These require again describing a right triangle with sides implied by the value given. For example,

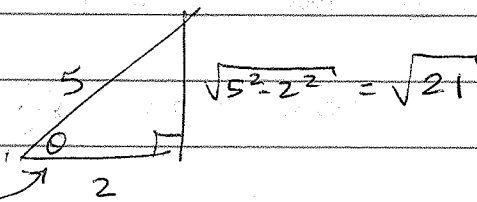
a) $\tan^{-1}(7)$ means a triangle whose tan ratio is $7/1$ would have missing

side $\sqrt{50}$, since the triangle \rightarrow has hypotenuse $\sqrt{1^2+7^2} = \sqrt{50}$



Thus, $\cos(\tan^{-1} 7) = \boxed{1/\sqrt{50}}$

b) $\sin(\cos^{-1}(\frac{2}{5}))$



$\boxed{\sin \theta = \sqrt{21}/5}$

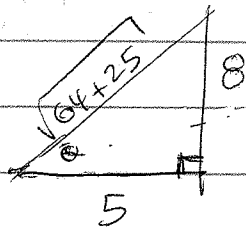
c) $\tan^{-1}(\tan(\pi/5)) = \boxed{\pi/5}$ (note, $\pi/5 \in \text{Dom } \tan^{-1}$)

d) $\cos^{-1}(\cos(-\pi/4)) = \cos^{-1}(\cos(\pi/4)) = \boxed{\pi/4}$

\uparrow
Not in
dom \cos^{-1}
but $\cos(-\pi/4)$
is defined

\uparrow
Equivalent
angle that
is in dom \cos^{-1}

e) $\cot(\tan^{-1}(8/5))$



$\cot \theta = 5/8$

We didn't need the hypotenuse!

Notice that dom \tan^{-1} is \mathbb{R}

Sec 8.6 con'd

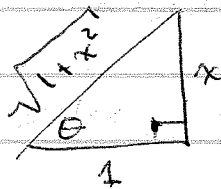
#7 a) $\sin(\arctan x) = \sin(\tan^{-1} x) = ?$

You don't need to rewrite this as I did, but it's more familiar notation, I think.

You need to create the right triangle whose $\tan \theta$ is x , that is, for which

$$\tan \theta = \frac{x}{1} \Rightarrow \tan^{-1}\left(\frac{x}{1}\right) = \theta$$

Thus,



is the triangle needed

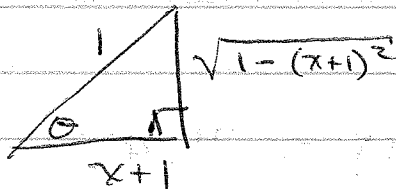
$\sin \theta$ is therefore $x / \sqrt{1+x^2}$

b) $\cos^{-1}(\cos x) = x$ ~~for $x \in [0, \pi]$~~
for $x \in [0, \pi]$, i.e., for $\cos x \in [-1, 1]$

c) $\tan(\underbrace{\cos^{-1}(x+1)}_{\theta})$

We need the right triangle for which $\cos \theta = \frac{x+1}{1}$, hence $\cos^{-1}\left(\frac{x+1}{1}\right) = \theta$

That is,



Hence, $\tan \theta = \frac{\sqrt{1-(x+1)^2}}{x+1} = \frac{\sqrt{-x^2-2x}}{x+1}$

#8) What's wrong with these?

$$a) \sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} \text{ so } \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{5\pi}{4}$$

Since $\frac{5\pi}{4} \in$ Restricted domain of sine on which the inverse is taken, then

$$\sin^{-1}(\sin x) \neq x, \text{ in this case } \frac{5\pi}{4}.$$

$$b) \tan^{-1}(\tan 5) = 5.$$

Again, the restricted dom of tangent on which \tan^{-1} is defined is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$5 \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, since $5 \text{ rad} > \frac{\pi}{2} \text{ rad}$.

$$c) \cos^{-1} x = \sec x$$

Wrong notation! $\cos^{-1} x \neq \frac{1}{\cos x} = \sec x$

$$d) \tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$$

In general, it is not the case that

$$f^{-1} = g^{-1} / h^{-1} \text{ when } f = g/h.$$

For example, $\frac{\sin^{-1}(1)}{\cos^{-1}(1)} = \text{undefined}$ since $\frac{\pi/2}{0}$ is undefined

$$\text{But, } \tan^{-1}(1) = \pi/4.$$