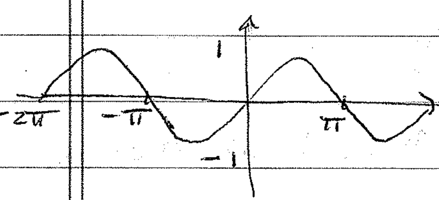
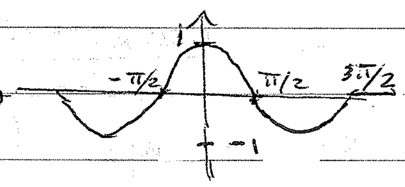
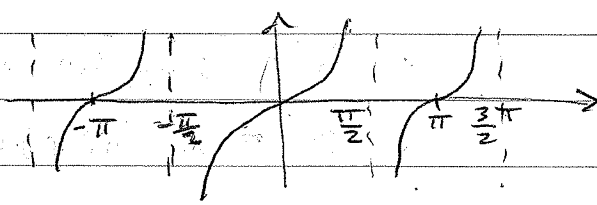
Sec. 8.5 – Graphing trigonometric fcns.

#1 Dom + range of $\sin x$, $\cos x$, $\tan x$ from graph

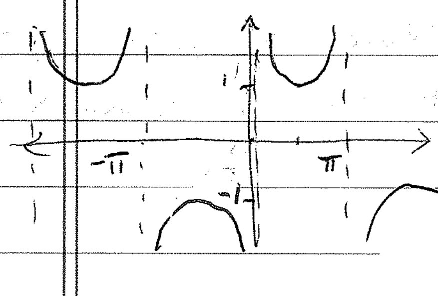


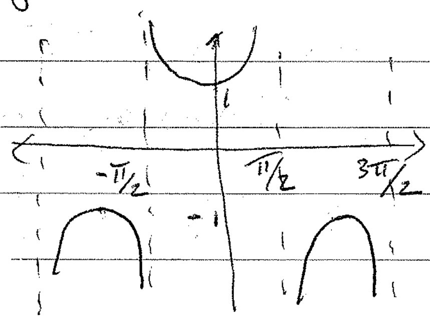


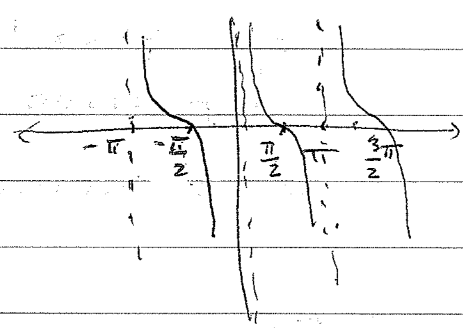


	$y = \sin x$	$y = \cos x$	$y = \tan x$
Dom	$\mathbb{R}$	$\mathbb{R}$	$x \neq \frac{\pi}{2} \pm n\pi$
Range	$[-1, 1]$	$[-1, 1]$	$\mathbb{R}$

Dom + range of $\csc x$, $\sec x$, $\cot x$ from graphs







	$y = \csc x = \frac{1}{\sin x}$	$y = \sec x = \frac{1}{\cos x}$	$y = \cot x = \frac{1}{\tan x}$
Dom	$x \neq n\pi$	$x \neq \frac{\pi}{2} \pm n\pi$	$x \neq n\pi$
Range	$(-\infty, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	$\mathbb{R}$

Notice how the reciprocal asymptotes are at values where parent fcn. = zero. It's good to see how one domain produces the other.

Then, memorize the shapes + intercepts + asymptotes — the dom + range follow.

#2 Using $\boxed{y = A\,\text{trig}(Bx + C) + D}$

where $\text{trig}\,\theta = \sin\theta$ or $\cos\theta$

we analyze transformations of sine + cosine according to these guidelines:

- A = amplitude = $\frac{\text{max} - \text{min}}{2}$
- D = vertical shift, up $(D>0)$ or down $(D<0)$
- $\frac{C}{B}$ = horizontal shift, right $\left(\frac{C}{B}<0\right)$ or left $\left(\frac{C}{B}>0\right)$
- Period = $\frac{2\pi}{B}$

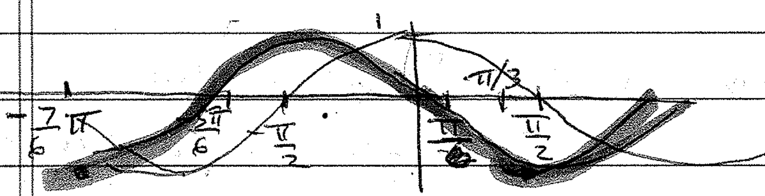
One important point: Because the period of $\sin x$ + $\cos x$ is 2π, the stretch or compression of transformations of sine + cosine is determined by $\frac{2\pi}{\text{period}} = B$.

But, for transformation of tangent fcn, since the period of $\tan x$ is π, the period of ~~the~~ tangent transformations is $\frac{\pi}{B}$, rather than $\frac{2\pi}{B}$.

8.5 #2

Using $y = A \text{ trig}(Bx + C) + D$

a) $g(x) = \cos\left(x + \frac{\pi}{3}\right)$ $\quad C = \frac{\pi}{3} > 0$ shifts left $\frac{\pi}{3}$



min at $x = \frac{-7\pi}{6} + \frac{\pi}{3}$

$\Rightarrow$ x-intercept $= \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

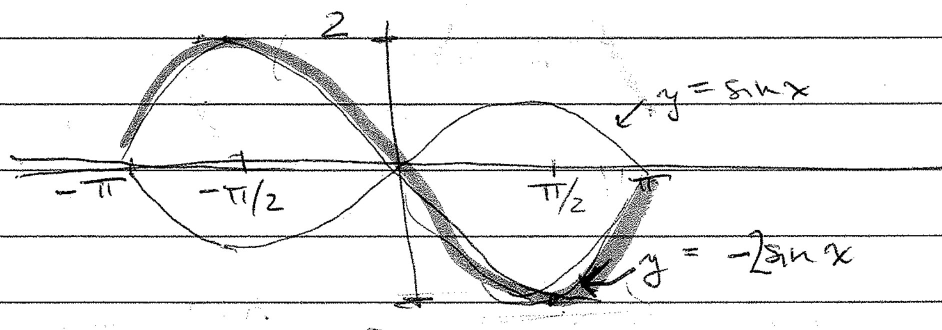
and $-\frac{\pi}{2} - \frac{\pi}{3} = \frac{-5\pi}{6}$

Where are the endpts?
On the graph above I've shown a min.
at $-\pi - \frac{\pi}{6} = -\frac{7}{6}\pi$ and at $\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

b) $g(x) = -2\sin(8x)$ $\quad A = \frac{\max - \min}{2} = \frac{2-(-2)}{2}$

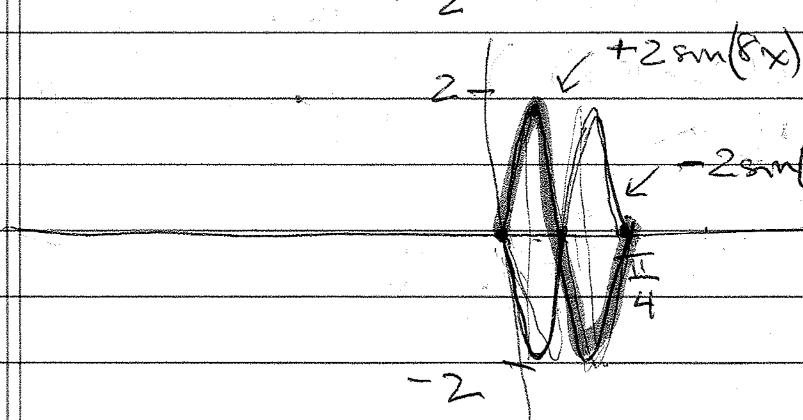
$\boxed{A = 2}$

$Bx = 8x, \quad P = \frac{2\pi}{B} = \frac{2\pi}{8} = \frac{\pi}{4}$ $\quad \boxed{P = \frac{\pi}{4}}$



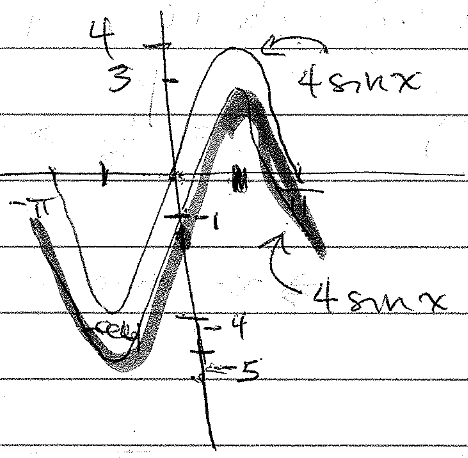
Compare to sin x period, which is 2π

It would help to look at the endpts so we can eyeball the new sh

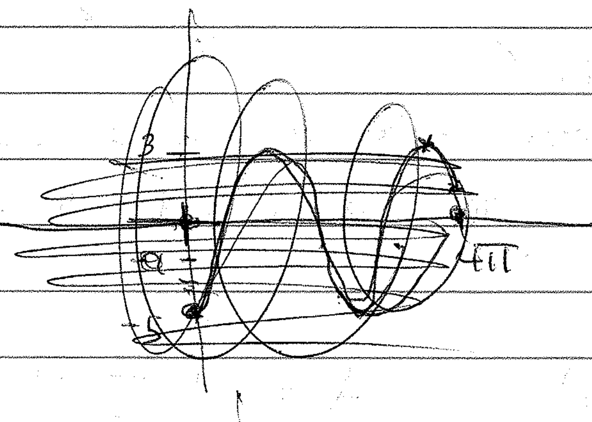


c) $g(x) = 4\sin\left(\frac{1}{2}x\right) - 1$

- $A = 4$
- $P = \frac{2\pi}{B} = \frac{2\pi}{1/2} = 4\pi$
- no shift ($C = 0$)



$4\sin x - 1$ ← Stretch this out so period is twice as long



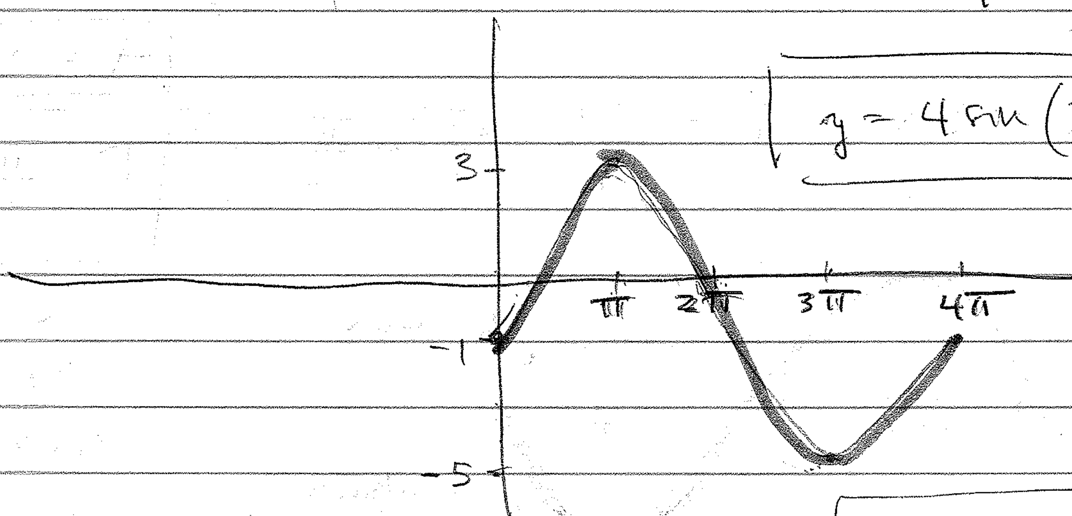
Endpts: $\frac{-C}{B}$ + $\frac{2\pi - C}{B}$

0 + $\frac{2\pi}{1/2} = 4\pi$

$g(0) = 4\sin(0) - 1$
$= -1$

$g(4\pi) = -1$

$\boxed{y = 4\sin\left(\frac{1}{2}x\right) - 1}$



d) $g(x) = \cos\left(2x - \frac{\pi}{3}\right) + 1$

(First graph $\cos(x - \pi/3)$ to get an idea of graph)

$$\boxed{\begin{array}{l} A = 1 \\ \text{shift} = \frac{\pi/3}{2} = \frac{\pi}{6} \text{ right} \\ \text{period} = \frac{2\pi}{2} = \pi \end{array}}$$