Sec 8.4

only sin is + (S)	(A) all trig fcns +
(T) only tan is +	(C) only cos is +

#1) Using the chart and the relationships of the trig fcns to their reciprocal fcns, decide which quadrant holds the terminal side of angle θ:

a) $\cos\theta > 0$, $\sin\theta < 0$ Q IV

b) ~~cot~~ $\tan\theta = 7$, $\cos\theta < 0$ Q III

c) $\sec\theta = 4$, $\sin\theta > 0$ QI

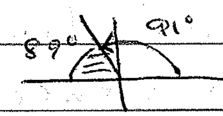
$\frac{1}{\cos\theta} = \frac{1}{4} > 0$ so $\cos\theta > 0$

d) $\sin\theta = \frac{12}{17}$, $\tan\theta < 0$ QII

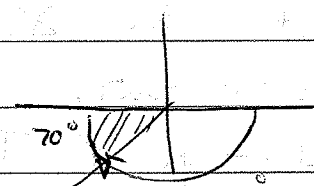
e) $\csc\theta < 0$, $\cot\theta > 0$ Q III

$\frac{1}{\sin\theta} < 0$, $\frac{1}{\tan\theta} > 0$

#2 a) $91^\circ \xrightarrow{\text{ref}} 89^\circ$



b) $-110^\circ \xrightarrow{\text{ref}} 70^\circ$



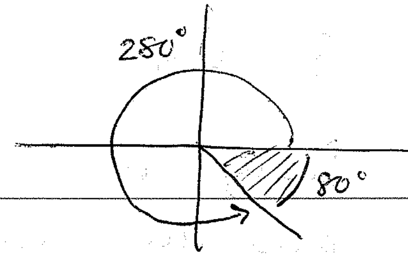
c) $1000^\circ \xrightarrow{\text{ref}} ?$

Find the remainder of 1000/360:

1000 ÷ 360 = 2, remainder 280.

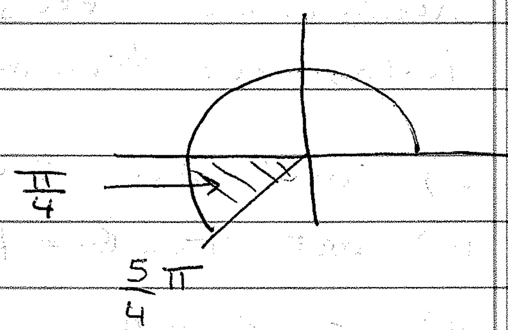
This tells us that the terminal side of θ after going twice around circle lands in QIV.

$280° \xrightarrow{ref} 80°$

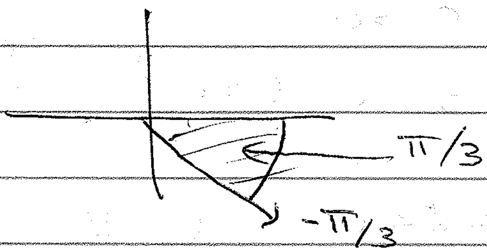


I've shaded the ref ∡ in a – c to indicate it is ~~...~~ simply an acute angle $(< 90°)$ above or below the x-axis.

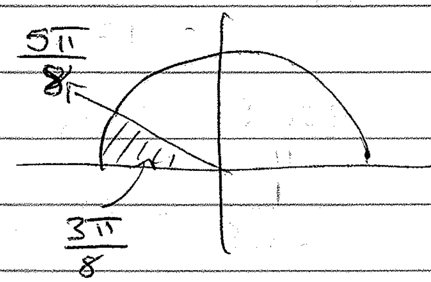
d) $\frac{5}{4}\pi \xrightarrow{ref} \frac{\pi}{4}$



e) $-\frac{\pi}{3} \xrightarrow{ref} \frac{\pi}{3}$

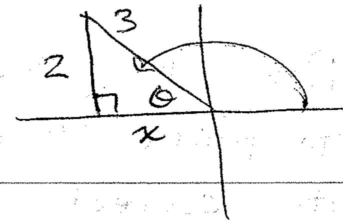


f) $\frac{5\pi}{8} \longrightarrow \frac{3\pi}{8}$



Don't look for a formula to get the reference angle; look at the circle + note the ∡ from x-axis, regardless of quadrant. Name the ref angle as positive always. This doesn't refer to its direction but just its acute angle measure.

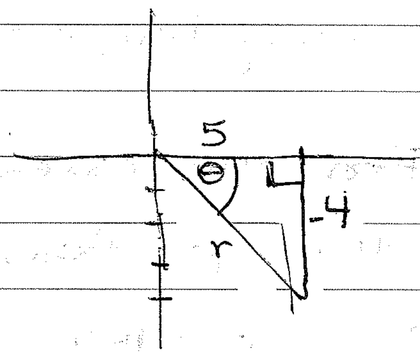
3) $\sin\theta = \frac{2}{3}$



$x = \sqrt{3^2 - 2^2}$

$x = \sqrt{5}$, but left of origin, so $\cos\theta = \frac{x}{3} = \frac{-\sqrt{5}}{3}$

<u>Another example:</u> $\tan\theta = \frac{-4}{5}$, θ in Q IV.



Find $\sin\theta$, $\cos\theta$, $\sec\theta$, $\csc\theta$, $\cot\theta$.

From $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$, only cosine (and its reciprocal, secant) are positive in Q IV

Find r: $r^2 = 5^2 + (-4)^2$

$= 25 + 16$

$= 40$

$r = \sqrt{40} = 2\sqrt{10}$

$\sin\theta = \frac{-4}{r}$

$\sin\theta = \frac{-4}{2\sqrt{10}} = \frac{-2}{\sqrt{10}}$

$\cos\theta = \frac{5}{2\sqrt{10}}$

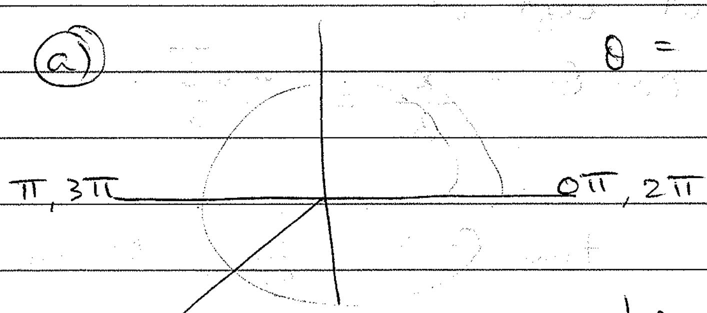
$\sec\theta = \frac{1}{\cos\theta} = \frac{2\sqrt{10}}{5}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{10}}{2}$

$\cot\theta = -\frac{5}{4}$

#4) Refer to Onyx's (or your) unit circle to place terminal side of angle θ in its correct quadrant:

(a) $\theta = \frac{13\pi}{4} = 3\pi + \frac{\pi}{4}$



$\frac{13\pi}{4}$ in Q III.

S	A
T	C

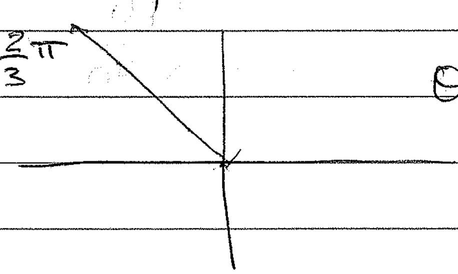
Only tan, cot are + here.

The ref angle is $\frac{\pi}{4}$. From the essential trig table the values of the 6 fcns are:

$\sin\frac{13\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$, $\csc\frac{13\pi}{4} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

$\cos\frac{13\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$, $\sec\frac{13\pi}{4} = -\sqrt{2}$ also

$\tan\frac{13\pi}{4} = \tan\frac{\pi}{4} = 1$, $\cot\frac{13\pi}{4} = \cot\frac{\pi}{4} = 1$

(b) 

θ is in Q II where only sine is positive (and csc)

S	A
T	C

The ref. angle is $\frac{\pi}{3}$

$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\csc\frac{2\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos\frac{2}{3}\pi = -\cos\frac{\pi}{3} = -\frac{1}{2}$, $\sec\frac{2}{3}\pi = -\sec\frac{\pi}{3} = -2$

$\tan\frac{2}{3}\pi = -\tan\frac{\pi}{3} = -\sqrt{3}$, $\cot\frac{2\pi}{3} = -\cot\frac{\pi}{3} = -\frac{1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

#4 c) $330°$ lies in Q IV, where only cosine is + (+ sec)
Ref. angle is $30°$.

$\sin 330 = -\sin 30 = -\frac{1}{2}$, $\csc 330 = -\csc 30 = -2$

$\cos 330 = \cos 30 = \frac{\sqrt{3}}{2}$, $\sec 330 = \sec 30 = \frac{2}{\sqrt{3}}$

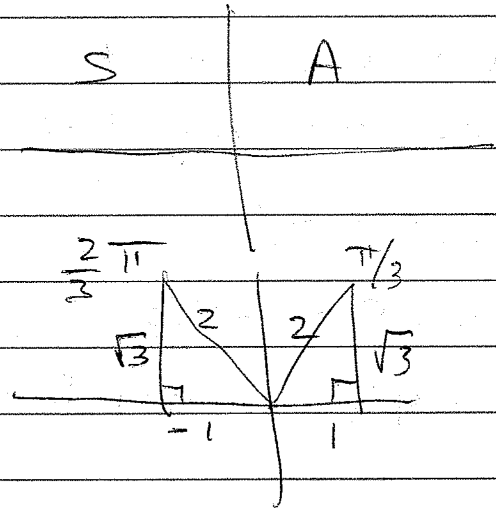
$\tan 330 = -\tan 30 = -\frac{1}{\sqrt{3}}$, $\cot 330 = -\cot 30 = -\sqrt{3}$

#5 a) $-210°$ lies in Q II, where only sine + csc are +. Ref angle is $30°$.
Find all 6 trig fcns.

b) 3π lies on x-axis, coterminal with π. Ref angle is $0°$
Find all 6 trig fcns.

c) $250°$ lies in Q III, where only tan + cot are +. Ref angle is $70°$.
Find all 6 trig fcns. (estimates)

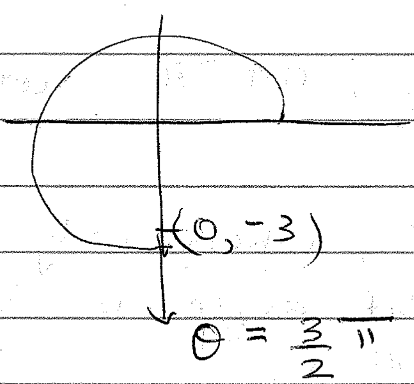
#6 $\sin\alpha = \sqrt{3}/2$ + $\sin\beta = \sqrt{3}/2$
What are α, β (must be in different quadrants, since they cannot be coterminal)



They'll lie in Q I + Q II
From essential trig table we know ref angle is $\frac{\pi}{3}$ in Q I.

In Q II the angle is $\frac{2\pi}{3}$.

#7) The pt $(0,-3)$ lies on y-axis, so we can't say the angle whose terminal side goes through is in std. position, as the book says.



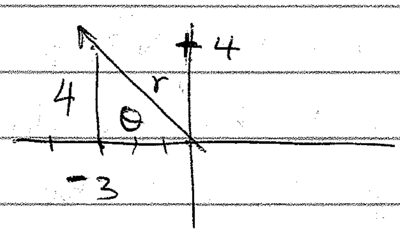
But you can easily see it's at 270°, or $\frac{3}{2}\pi$. The ref. angle is $\frac{\pi}{2}$, and ~~same~~ the fns. are like this:

$\sin \frac{3}{2}\pi = -\sin \frac{\pi}{2} = -1 \quad (\csc \frac{\pi}{2} = -1)$

$\cos \frac{3}{2}\pi = \cos \frac{\pi}{2} = 0 \quad (\sec \frac{\pi}{2} \text{ undefined})$

$\tan \frac{3}{2}\pi = -\tan \frac{\pi}{2}$ undefined $(\cot \frac{\pi}{2} = 0)$

#8) Sketch this to see where θ lies:



$r^2 = 3^2 + 4^2 = 25$

$r = 5$

S	A
T	C

sin + csc are +

Others are negative

$\sin\theta = 4/5, \quad \csc\theta = 5/4$

$\cos\theta = -3/5, \quad \sec\theta = -5/3$

$\tan\theta = 4/-3 = -4/3, \quad \cot\theta = -3/4$

#9. Because the terminal side intersects the unit circle, $r = 1$. To find x & y, you have to find r of the larger right triangle with sides 3 & 5, then scale it down by this value, as shown.

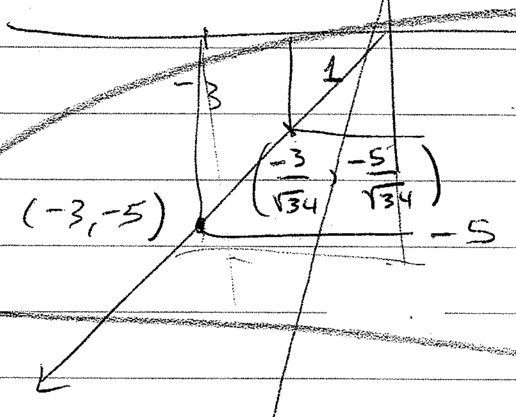
$$a^2 + b^2 = 3^2 + 5^2 = 9 + 25 = 34 = r^2$$

$$a^2 + b^2 = r^2$$

$$(-3)^2 + (-5)^2 = 34$$

$$\frac{(-3)^2}{34} + \frac{(-5)^2}{34} = \frac{34}{34} = 1$$

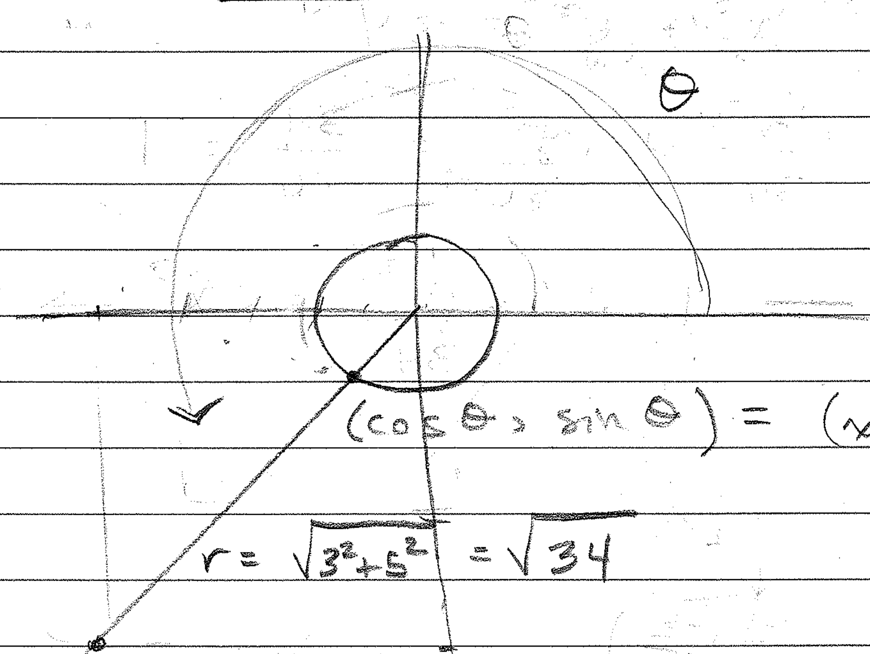
$$\frac{a^2}{34} = x^2 \longrightarrow x =$$



Sec 8.4 #9

An angle in std position (counterclockwise starting at O π) goes through the pt ~~(-3, 4)~~ (-3, -5).

What are the coordinates of the pt of intersection of the terminal side of this angle with the unit circle?



$(\cos\theta, \sin\theta) = (x, y)$ on unit circle

$r = \sqrt{3^2+5^2} = \sqrt{34}$

$(-3,-5) = (r\cos\theta, r\sin\theta) = (x, y)$ on terminal side of angle θ

We don't need to find θ to answer the question. We need only r. From the Pyth. theorem:

$$x^2+y^2 = (-3)^2+(-5)^2 = 9+25 = 34$$

$$r = \sqrt{34}$$

Thus, $(-3,-5)$ lies on a circle of radius $\sqrt{34}$.

The corresponding pt on the circle of radius 1 (the unit circle) is simply $\left(\frac{-3}{\sqrt{34}}, \frac{-5}{\sqrt{34}}\right)$ from the relationships

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{34}}$$

$$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{34}}$$

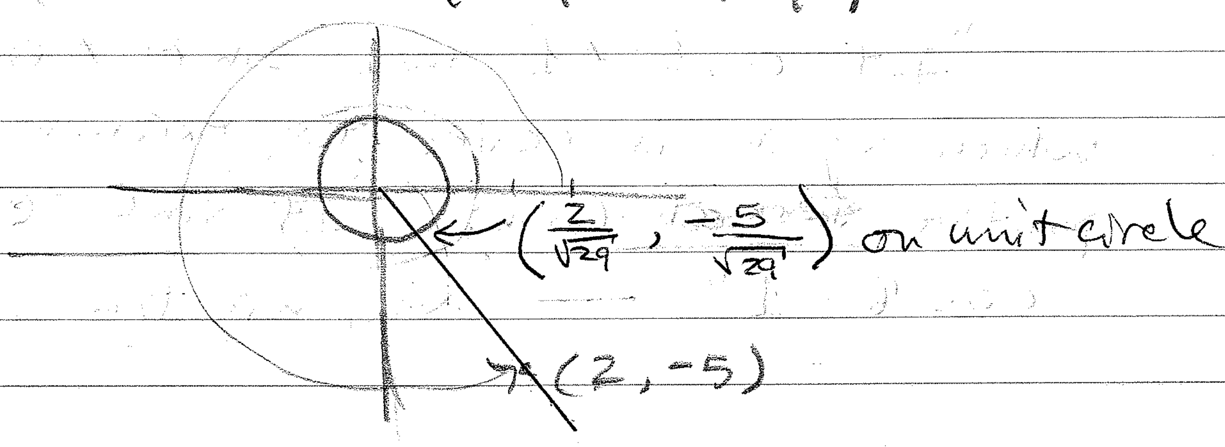
In general, the pt. on the unit circle corresponding to a pt (a,b) that an angle in std position goes through is

$$\left(\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}\right)$$

<u>Ex</u> Find the coordinates of the pt on the unit circle that intersects the terminal side of an angle through $(2,-5)$?

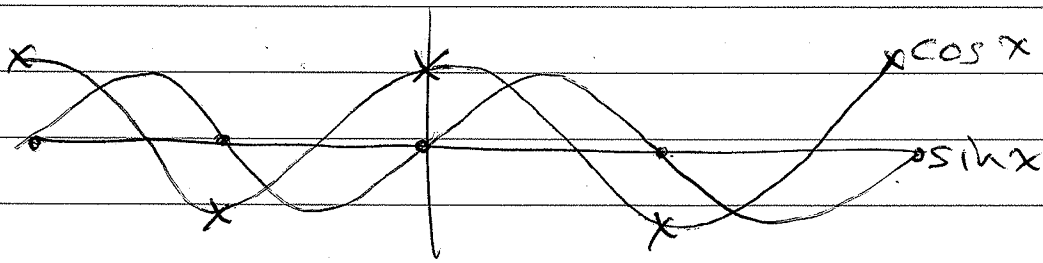
<u>Soln</u> $r = \sqrt{2^2+(-5)^2} = \sqrt{29}$

$$(x,y) = \left(\frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\right)$$



#10) "If $\sin\theta \neq 0$ then $\cos\theta \neq 1$."

Look at the graphs of $\sin x$ + $\cos x$ to analyze this statement.



The dots show where $\sin\theta = 0$. ~~the~~ ~~need~~ The exs where $\cos\theta = 1$.
There are 3 places where $\cos\theta = 1$ on my sketch. At each of these, $\sin\theta = 0$.
It's easier to examine the ~~contrapositive~~ contrapositive (equivalent) statement:
"If $\cos\theta = 1$ then $\sin\theta = 0$"
This is clearly true.

However, if $\sin\theta = 0$ then the $\cos\theta$ is not necessarily $= 1$. Sometimes it's -1.
So, the converse is false:
"If $\cos\theta \neq 1$ then $\sin\theta \neq 0$"
which again is easier to examine in contra-positive ~~format~~ form: "If $\sin\theta = 0$ then $\cos\theta = 1$" — No, not true.