## Ch. 6.4 #1

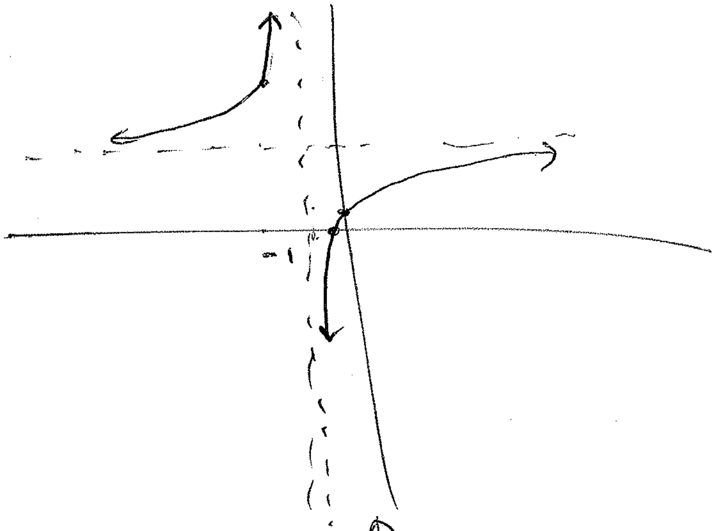
a) $f(x) = \dfrac{3x+1}{x+1}$

VA: $x = -1$

HA: $y = 3/1 = 3$ since $n = m$

$f(0) = 1$ (y-int)

$3x + 1 = 0$ at $x = -1/3$ (root)




b) $f(x) = \dfrac{8x+7}{12x-5}$

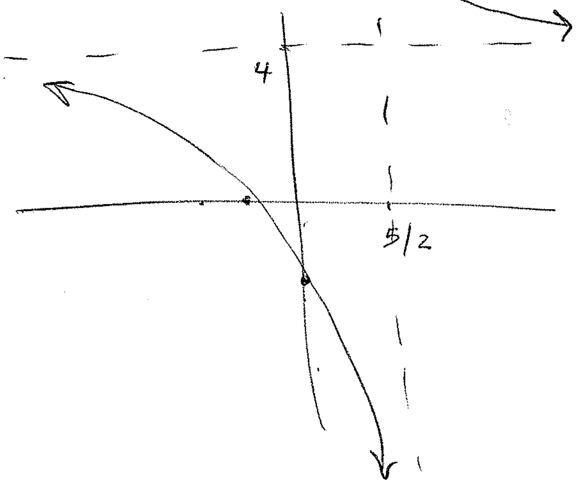
VA: $x = 5/2$

HA: $y = 4$ since $n = m$

$f(0) = -7/5$ (y-int)

$8x + 7 = 0$ at $x = -7/8$ (root)

I didn't check pts other than intercepts.




c) $f(x) = \dfrac{x^2 \cancel{(x-3)}}{(x+2)(x-2)\cancel{(x-3)}}$

Dom: $x \neq \pm 2, 3$

Hole: at $x = 3$, $f(x) = 9/5$

V.A.: $x = 2$, $x = -2$

y-int: $f(0) = 0$

root: $x = 0$

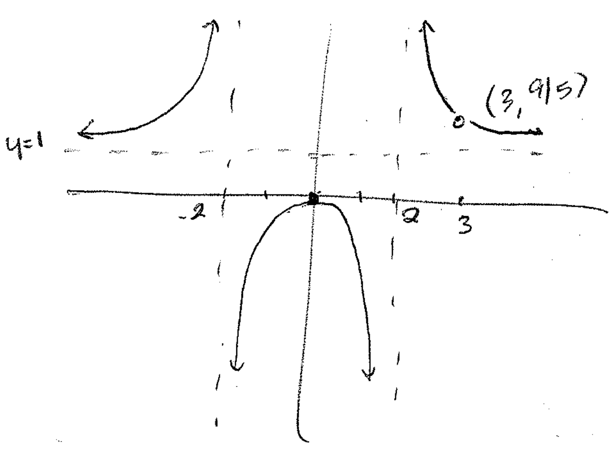
Because $n = m$, HA is $y = 1/1 = 1$

Inspect at $x = 1, -1, 3, -3$

$f(1) = -1/3$ $\quad f(3) = 9/5$ (hole)

$f(-1) = -1/3$ $\quad f(-3) = 9/5$

It's an even fcn $(f(-x) = f(x))$



Ch. 6.4

#1d) $f(x) = \dfrac{x^2-2x-3}{x^3-5x^2-6x} = \dfrac{(x-3)\cancel{(x+1)}}{x(x-6)\cancel{(x+1)}}$

Dom: $x \neq 0, 6, -1$

But $x=0$ + $x=6$ are VA

while $x=-1$ is a hole where

$f(-1) = \dfrac{(-1-3)}{-1(-1-6)} = \dfrac{-4}{7}$

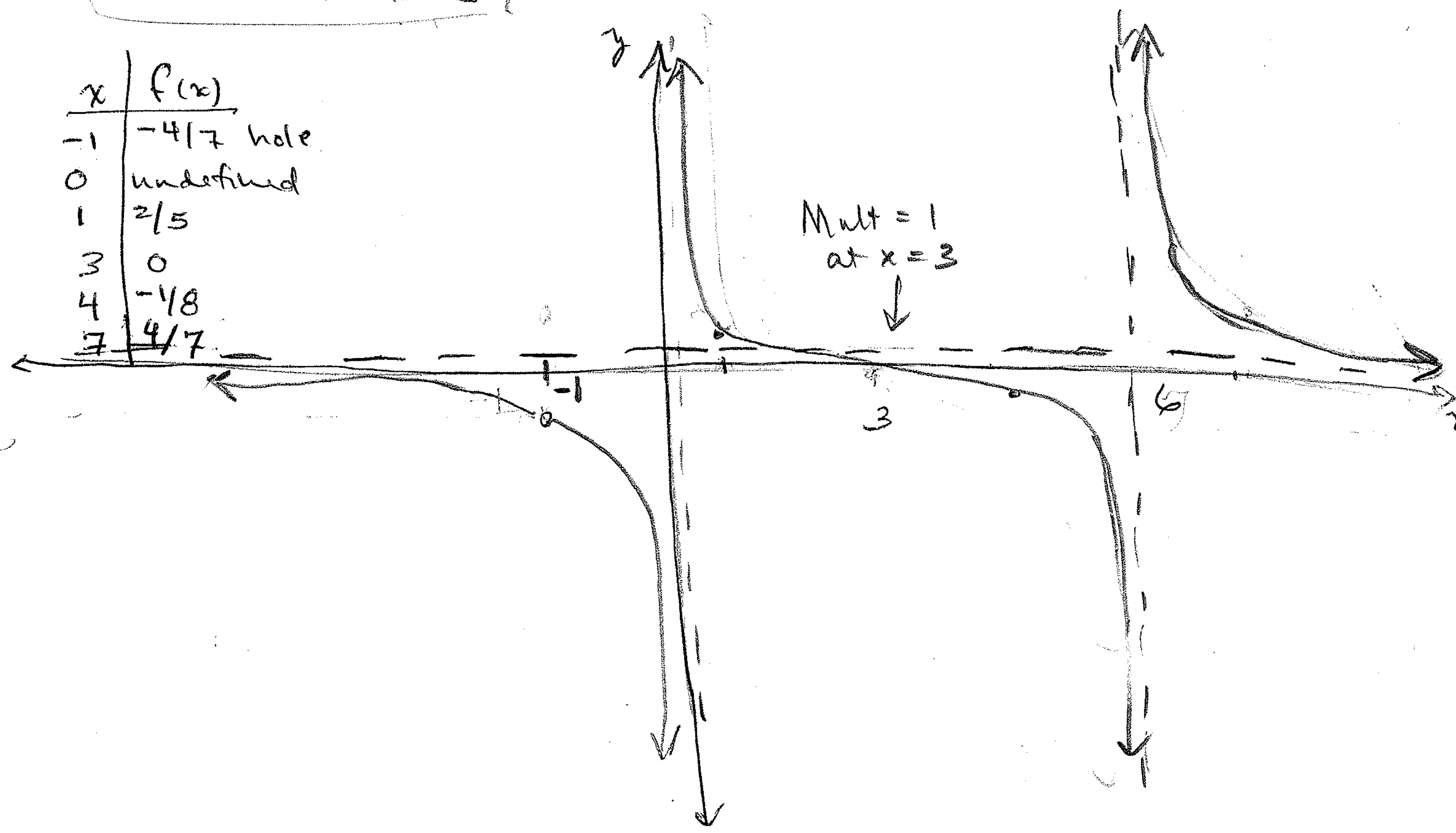
Caution: Substitute the value into the simplified fcn.

$m > n$ so there's an HA at $y=0$

So far: $\boxed{\text{VA: } x=0,\ x=6}$ $\boxed{\text{HA: } y=0}$

$\boxed{\text{root: } x-3=0 \text{ or } x=3}$ $\boxed{\text{hole at } (-1, -4/7)}$

| $x$ | $f(x)$ |
|---|---|
| $-1$ | $-4/7$ hole |
| $0$ | undefined |
| $1$ | $2/5$ |
| $3$ | $0$ |
| $4$ | $-1/8$ |
| $7$ | $4/7$ |



## Chapter 6 Problem 1 (con'd)

☆ #1(e) $\dfrac{x^3}{4x^2-4} = f(x)$

$(2x-2)(2x+2)$

$$f(x) = \frac{x^3}{4(x+1)(x-1)}$$

Dom: $4x^2-4 \neq 0$

$x \neq \pm 1$

| | |
|---|---|
| VA: | $x = \pm 1$ |
| HA: | none b/c $n > m$ |
| SA: | yes, b/c $n = m+1$ |

To find SA, do $\div$

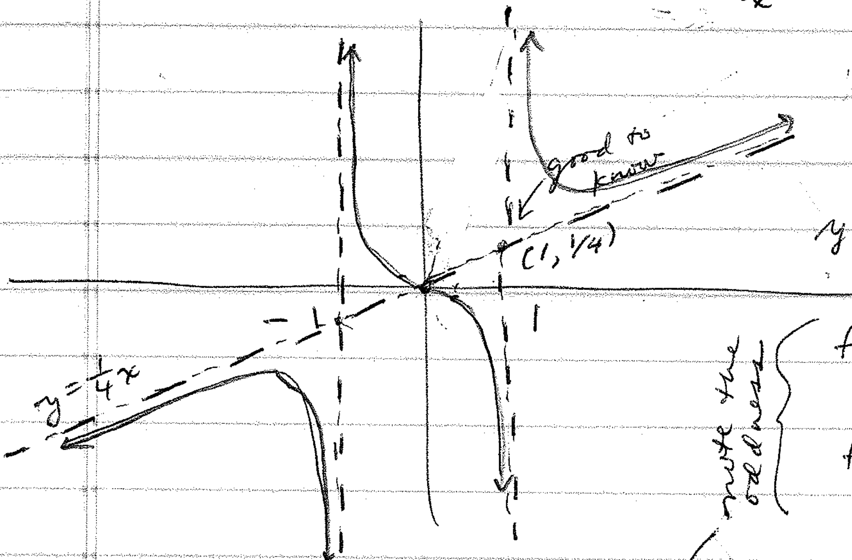
$$4x^2-4 \overline{\big)\, x^3} \quad \frac{1}{4}x + \frac{x}{4x^2-4}$$

$$\underline{-(x^3 - x)}$$

$$x$$

SA: $y = \frac{1}{4}x$

(note the intersection of VA + SA)




Points?

y-int: $f(0) = 0$

root $x = 0$

note the oddness

$f(1/2) = +1/8 / -3 = -1/24$

$f(-1/2) = -1/8 / -3 = 1/24$

oddness $\Rightarrow$ symmetry across the origin

Note We inspect $+\infty$ + $-\infty$ in the original fcn to find $f(x) \to +\infty$ as $x \to \infty$

$\dfrac{x^3}{4x^2-4}$ + $f(x) \to -\infty$ as $x \to -\infty$

# Ch. 6.4 #1 continued

#1f) $f(x) = \dfrac{x^3+1}{x^2+x} = \dfrac{\cancel{(x+1)}(x^2-x+1)}{x\cancel{(x+1)}}$, Dom $x \neq 0, -1$

~~VA: x = −1~~

$\boxed{\begin{array}{l}\text{VA: } x = 0 \\ \text{hole: } x = -1\end{array}}$

$n = m + 1 \rightarrow$ SA − find by long $\div$

OK to $\div$ the simplified form of $f(x)$

$$\begin{array}{r} x - 1 + \tfrac{1}{x} \\ x \,\overline{)\, x^2 - x + 1} \\ -(x^2 \qquad\quad \\ \hline -x + 1 \\ -(-x \qquad) \\ \hline 1 \end{array}$$

$\boxed{\text{SA } y = x - 1}$

$f(0) =$ undefined (no y-int; the VA is $x = 0$)

$f(x) = 0 \longrightarrow x^3 + 1 = 0 \longrightarrow$ ~~not exact factor (x³~~

$x^3 = -1 \rightarrow \boxed{x = -1}$

But that's a hole!

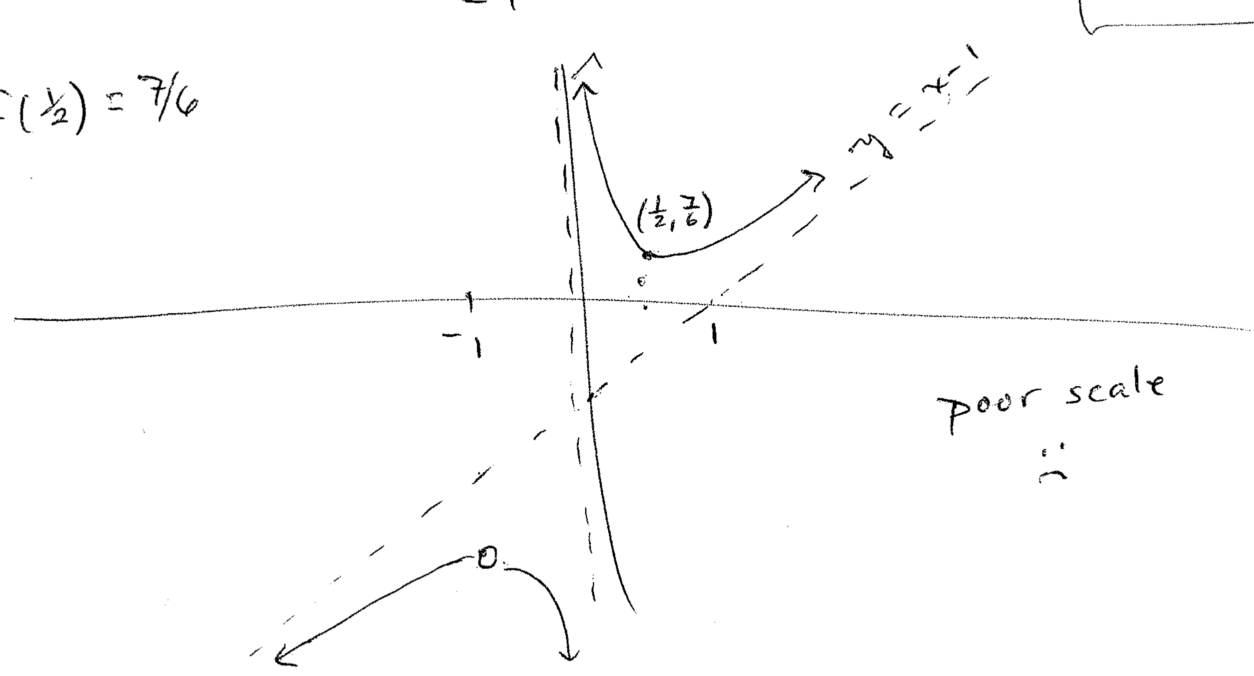
Finally, find value of $f(x)$ at the hole:

$f(-1)$ of the simplified form:

$f(-1) = \dfrac{(-1)^2 - (-1) + 1}{-1} = \dfrac{3}{-1} = -3$

$\boxed{\begin{array}{c}(-1, -3) \\ \text{hole}\end{array}}$

Eyeball $f(\frac{1}{2}) = 7/6$




poor scale

:(

Ch. 6.4 #1 continued

#1g) $f(x) = \dfrac{-(x-1)^3(x+4)}{(x+2)^2}$

Dom: $x \neq -2$

$\boxed{\text{VA: } x = -2}$ (no cancelling)

$n = 4$, $m = 2$

$n > m \rightarrow$ no HA

$n \neq m+1 \rightarrow$ no SA

A word about behavior at $\pm\infty$. Inspect the sign of $f(x)$ at $x \to +\infty$ to see $\dfrac{(-)(+)(+)}{(+)} = -$.

Inspect sign at $x \to -\infty$ to see $\dfrac{(-)(-)(-)}{(+)} = -$

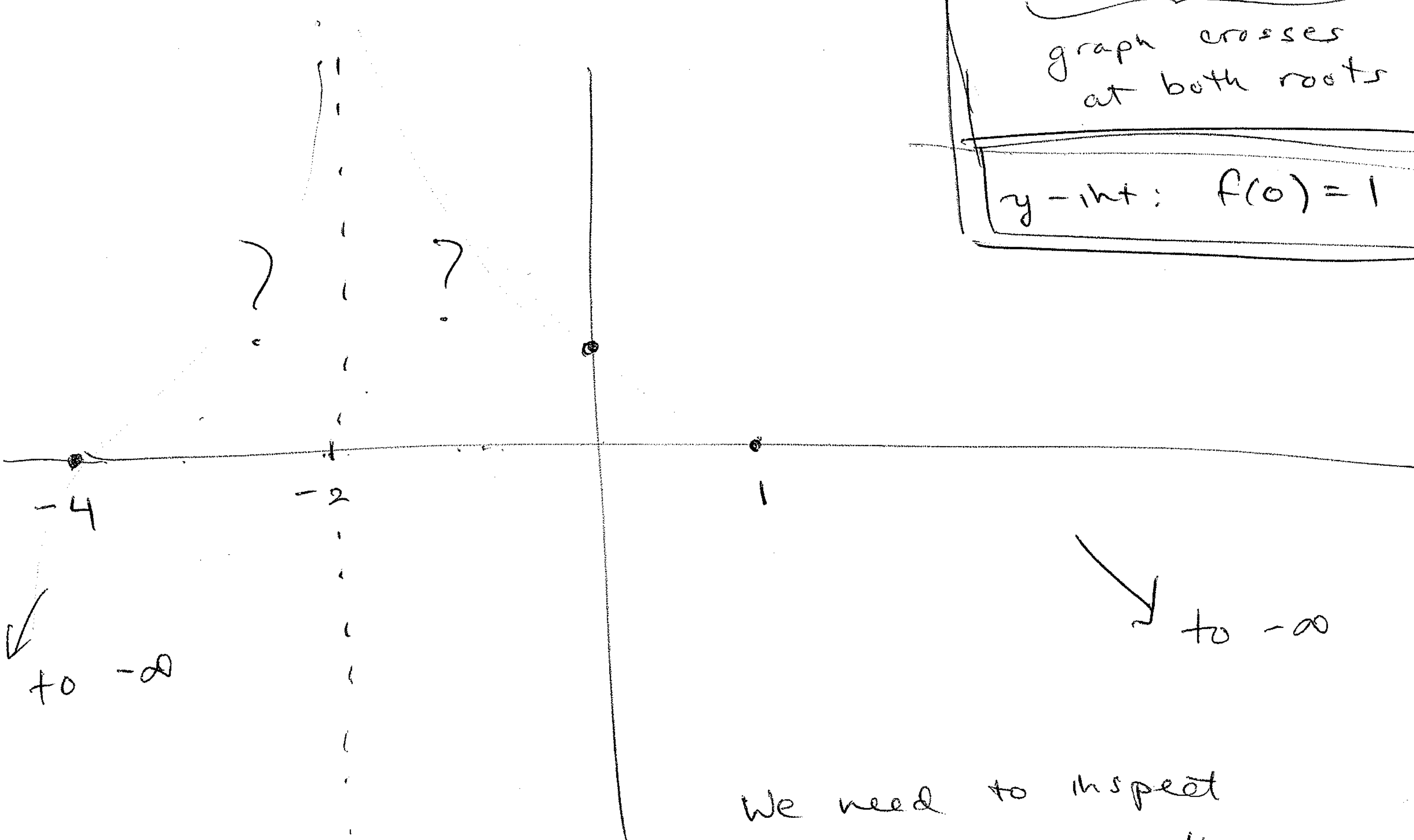
Roots: $f(x) = 0$ when $-(x-1)^3(x+4) = 0$

$x = 1, \; -4$

mult: 3  mult: 1

graph crosses at both roots

y-int: $f(0) = 1$




We need to inspect as $x \to -2$ on the right + left

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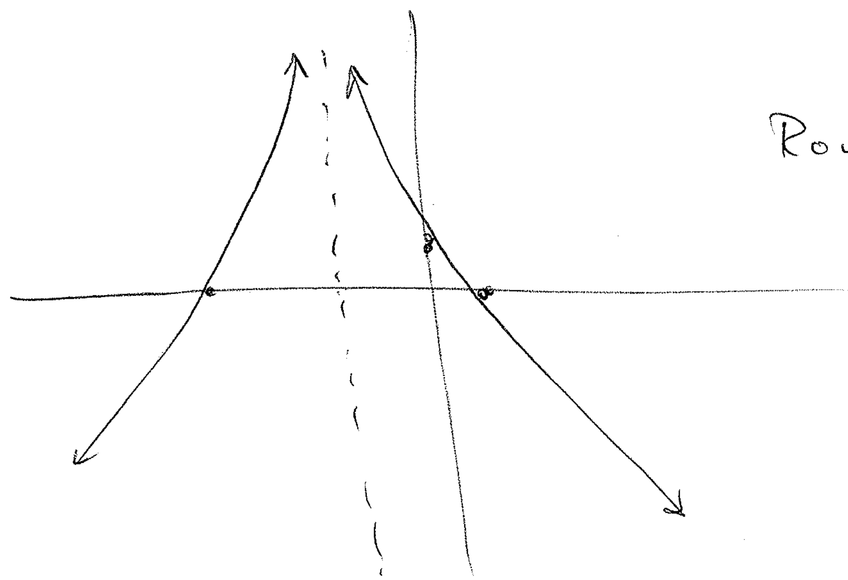
$f(-3) = 64$, $f(-1) = 24$



Rough is okay.

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#1h) $f(x) = \dfrac{(x+1)^3(x-2)x}{(x+1)(x-2)^3 x}$

Clearly there's cancellation of 2 factors, hence 2 holes.

Dom: $x \neq -1, +2, 0$
VA: $x = +2$
holes at: $x = -1, 0$

Simplify $f(x) = \dfrac{(x+1)^2}{(x-2)^2}$

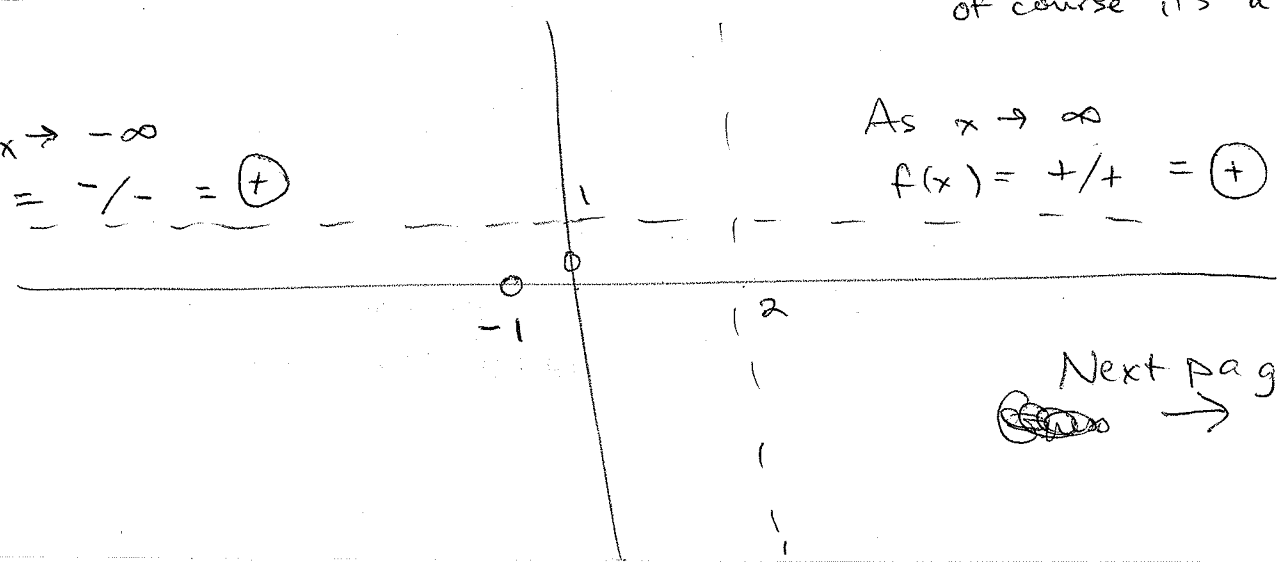
$n = m \longrightarrow$ HA: $y = 1/1$ or $\boxed{y = 1}$

y-int ?
$f(0) =$ undefined (hole)
so calculate $f(0)$ of simplified fcn: $f(0) = 1/4$ (hole); Also $f(-1) = 0$ (hole)

roots? $f(x) = 0 = (x+1)^2 \longrightarrow x = -1$ mult of 2 but of course it's a hole

As $x \to -\infty$
$f(x) = -/- = +$

As $x \to \infty$
$f(x) = +/+ = +$

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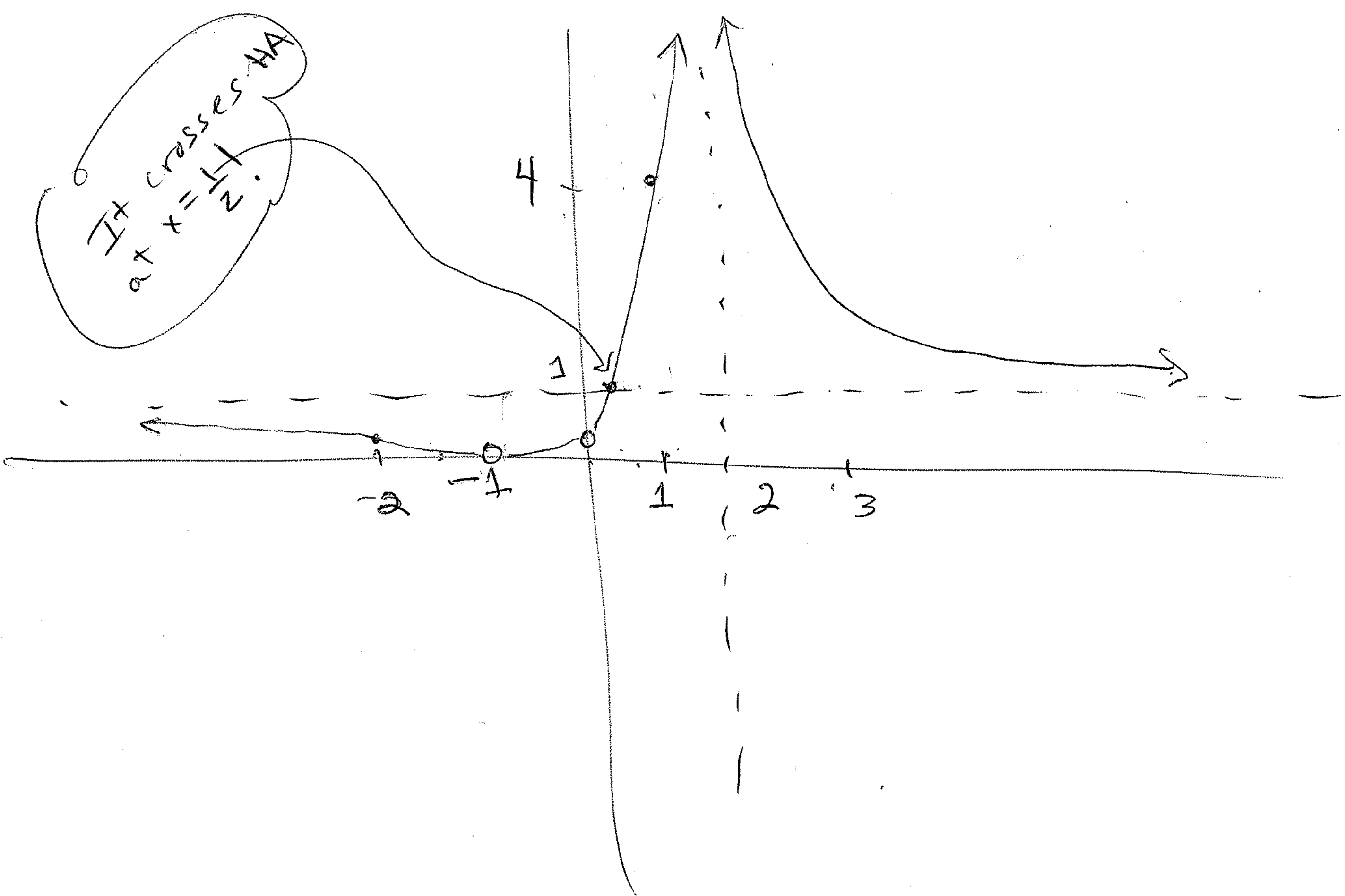
We need some more pts.

$$f(-2) = \frac{(-2+1)^2}{(-2-2)^2} = \frac{1}{16}$$

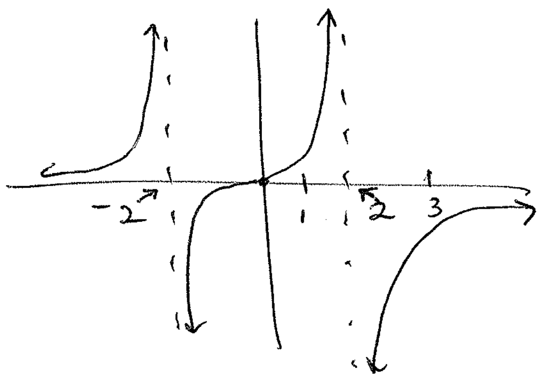
$$f(1) = \frac{(1+1)^2}{(1-2)^2} = \frac{4}{1} = 4$$

$$f(3) = \frac{(3+1)^2}{(3-2)^2} = \frac{16}{1} = 16$$



Ch. 6.4 # 2a + b

(a) $f(x) = \dfrac{-x}{x^2-4}$




$\boxed{\begin{array}{l} \text{Dom: } x \neq \pm 2 \\ \text{V.A: } x = 2, \quad x = -2 \\ \text{H.A: } n < m \\ \quad \text{so } y = 0 \text{ is HA} \end{array}}$

$\boxed{f(0) = 0 \text{ (y-int and root)}}$

Inspect between asymptotes:

As $x \to 2^-$ ($x$ goes to 2 on the left – NOT -2). Is $f(x) > 0$ or $< 0$? It's negative / negative = positive.

(Check: $f(1) = -1/-3 = 1/3$)

As $x \to 2^+$ ($x$ goes to 2 on the right) is $f(x) > 0$ or $< 0$? It's negative / positive = negative.

(Check: $f(3) = -3/5$)

| $x$ | $f(x)$ |
|---|---|
| -3 | -3/5 |
| -2 | undefined |
| -1 | 1/3 |
| 0 | 0 |
| 1 | 1/3 |
| 2 | undefined |
| 3 | -3/5 |

Notice that $f(x)$ is odd:

$$f(-x) = \frac{-(-x)}{(-x)^2+4} = \frac{x}{x^2+4} = -f(x)$$

Symmetric on the origin.

# 2 a + b continued

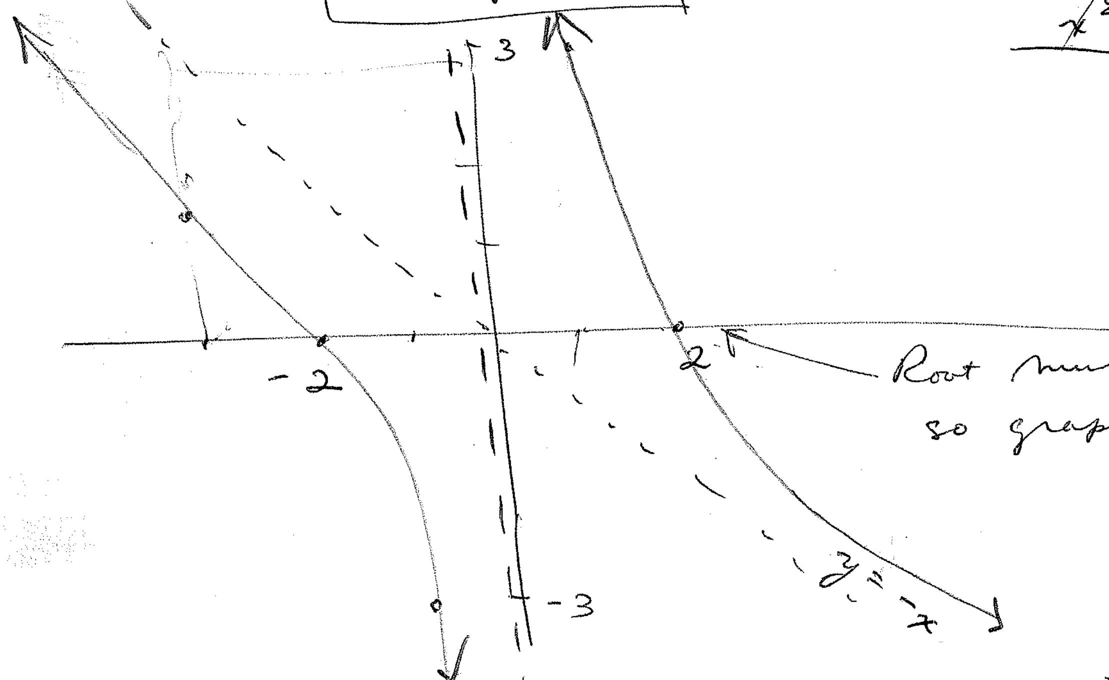
(b) $f(x) = \dfrac{x^2 - 4}{-x}$

$\boxed{\begin{array}{ll} \underline{\text{Dom}}: \ x \neq 0 & \underline{\text{roots}}\ x = \pm 2 \\ \underline{VA}: \ x = 0 & (x^2 - 4 = 0) \end{array}}$

$n = m + 1 \Rightarrow SA.$

$$\begin{array}{r|l} & -x + 4/x \\ -x & \cancel{x^2} - 4 \\ & \underline{\cancel{x^2}} \quad \downarrow \\ & \quad -4 \end{array}$$

$\boxed{\underline{SA}: \ y = -x}$




Root multiplicity is odd so graph crosses.

Suspect as $x \to \infty$ & as $x \to -\infty$

$f(x) \to -\infty$ $\qquad$ $f(x) \to \infty$

Check other pts: $f(-1) = \dfrac{1-4}{1} = -3$, $f(+1) = \dfrac{1-4}{-1} = -3$

$f(-3) = \dfrac{9-4}{3} = \dfrac{5}{3}$, $f(+3) = \dfrac{9-4}{-3} = \dfrac{-5}{3}$

In fact $-f(x) = f(-x)$, $\underline{\text{so fcn. is odd.}}$

Is $f(-3)$ above SA $y = -x$? No, since

$f(-3) = 5/3$ & $y = -x = -(-3) = 3$

# Ch. 6.4 Problems 3 + 4

Sketch a graph with the characteristics. Then write an equation for a fcn. that meets these characteristics. There are more answers than one + graphs may vary. The idea is to account for all known elements:

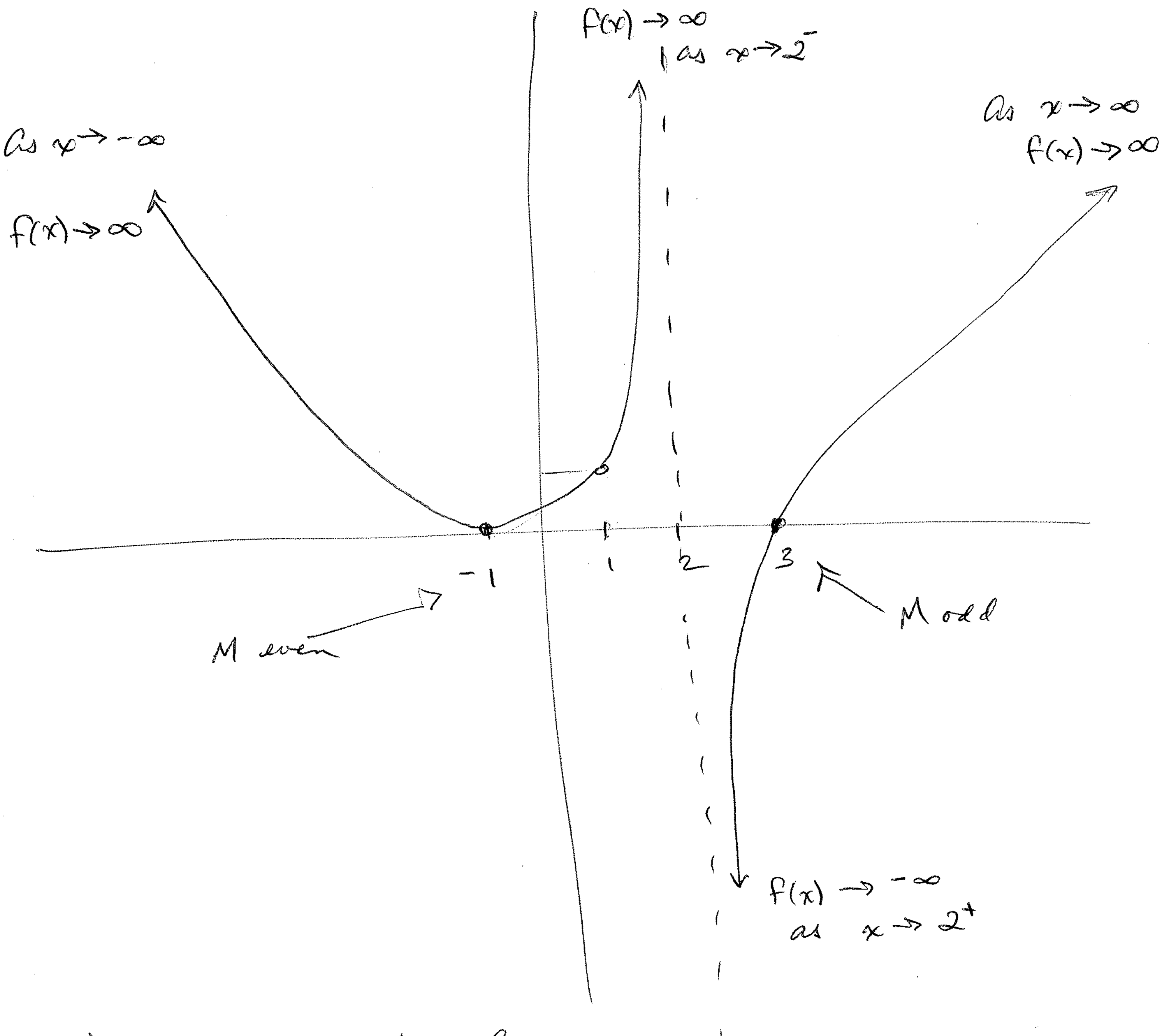
- Dom: $x \neq 1, 2$
- Hole at $(1,1)$

$\Big\}$ So $x = 1$ came from a factor that cancelled.

- VA: $x = 2$ $\}$ So $x = 2$ came from a factor that did not cancel.

- As $x \to 2^-$, $f(x) \to \infty$ This means as you approach $x=2$ from the left, the fcn shoots up.
- As $x \to 2^+$, $f(x) \to -\infty$ This means as you approach $x = 2$ from the right, the fcn shoots down.
- Roots $x = -1$, $x = 3$ So the numerator factors as $(x+1)(x-3)$ but possibly with multiplicities $\geq 1$.
- As $x \to +\infty$, $f(x) \to +\infty$
  As $x \to -\infty$, $f(x) \to +\infty$

$\Big\}$ Both ends tend up




Build a possible fcn. from these elements. I'll add here that there is no HA, so $n > m$. There is no SA, so $n > m+1$, too.

First try: $f(x) = \dfrac{(x-1)(x-3)(x+1)}{(x-1)(x-2)}$

roots w mult = 1 each. but graph shows $x = -1$ touches, so mult has to be even.

cancel

asymptote

- Note that $(x+1)$ has to have a power of 2, 4, ...
  & $(x-3)$ a power of 1, 3, 5, ...

- Since $n > m+1$ we'll need another factor on top. Let it be $(x+1)$ since $x = -1$ is a "double root.

Second try : $$f(x) = \frac{(x-1)(x-3)(x+1)^2}{(x-1)(x-2)} \qquad \begin{matrix} n=4 \\ m=2 \end{matrix}$$

You could multiply by a constant without violating the characteristics.

NOT IN BOOK

$$f(x) = \frac{2(x^2-1)(x-4)^2}{(x-2)^4}$$

$\boxed{\begin{array}{l} \text{Dom: } x \neq 2 \\ \text{VA: } x = 2 \end{array}}$

$\boxed{y\text{int}: f(0) = \frac{-32}{16} = -2}$

$\boxed{\begin{array}{l} \text{roots: } 2(x^2-1)(x-4)^2 = 0 \\ \text{at } x = \pm 1, 4 \end{array}}$

$m = n = 4$ so

we have a HA of $\boxed{y = \frac{a_m}{b_n} = \frac{2}{1} = 2 \text{ HA.}}$

| $x$ | $f(x)$ |
|---|---|
| 2 | undefined |
| 1 | 0 |
| 0 | -2 |
| -1 | 0 |
| -2 | 9/8 |
| 3 | 1 |