Sec. 5.3 HW Solutions

#1) Factoring (review of Ch. 2, but now we solve for the roots of the fcn. $f(x)$)

a) $x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5, -3$

b) $2x^2 - x = 1$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = -1/2, 1$

c) $-x^2 - x + 12 = 0$

$-(x^2 + x - 12) = 0$

$-(x-3)(x+4) = 0$

$x = 3, -4$

d) $8x^2 + 14x - 4 = 0$

$4x^2 + 7x - 2 = 0$

$(4x-1)(x+2) = 0$

$x = 1/4, -2$

e) $x^2 - 5 = 0$

$(x+\sqrt{5})(x-\sqrt{5}) = 0$

$x = \pm\sqrt{5}$

(A difference of 'squares' though not perfect squares)

$\longrightarrow$ Before, we'd set $x^2 =$ constant and solve, as in

$\sqrt{x^2} = \sqrt{5}$

$x = \pm\sqrt{5}$

$\Rightarrow$ The right side method is still fine, but in the context of factoring as $(x-r_1)(x-r_2)$, we use the left side method.

#2 a) $9x^2 - 16 = 0$

$9x^2 = 16$

$x^2 = 16/9$

$x = \pm\sqrt{16/9} = \pm 4/3$

b) $2x^2 - 6x = -3$

$2(x^2 - 3x) = -3$

$2\left(x^2 - 3x + (-3/2)^2\right) = -3 + 2\cdot(-3/2)^2$

$(x - 3/2)^2 = \dfrac{-3 + 9/2}{2}$

$(x - 3/2)^2 = \dfrac{3}{4} \longrightarrow$

$$x - 3/2 = \pm\sqrt{3/4}$$

$$x = \frac{3}{2} \pm \sqrt{3/4} = \frac{3 \pm \sqrt{3}}{2}$$

(Check by quadratic formula: $f(x) = 2x^2 - 6x + 3 = 0$)

$$x = \frac{6 \pm \sqrt{36 - 24}}{4} = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$x = \frac{3 \pm \sqrt{3}}{2}$$

c) $4x^2 + 8x - 52 = 0$

$1x^2 + 2x - 13 = 0$

$x^2 + 2x + 1 = 13 + 1$

$(x+1)^2 = 14$

$x = -1 \pm \sqrt{14}$

2 roots: $-1 + \sqrt{14}$

$-1 - \sqrt{14}$

$b^2 - 4ac$

$2^2 - 4(1)(-13)$

$= 4 + 52$

$= 56 > 0$

Check w/ QF:

$$x = \frac{-2 \pm \sqrt{4 - 4(-13)}}{2}$$

$$= \frac{-2 \pm \sqrt{56}}{2}$$

$$x = -1 \pm \sqrt{14}$$

d) $x^2 + 4x + 1 = 0$

$x^2 + 4x + 4 = -1 + 4$

$(x+2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x = -2 \pm \sqrt{3}$

e) $x^2 = 21 - 4x$

$x^2 + 4x = 21$

$x^2 + 4x + 4 = 25$

$(x+2)^2 = 25$

$x + 2 = \pm 5$

$x = -2 \pm 5$

$x = 3, -7$

(We could have factored the equation!)

#6) The number of roots a quadratic equation has is determined as follows:

i) $b^2-4ac>0 \Rightarrow 2$ roots

ii) $b^2-4ac=0 \Rightarrow \underline{1}$ root with multiplicity 2

iii) $b^2-4ac<0 \Rightarrow$ no roots

We sometimes refer to situation (ii) as a double root.

#7) $f(x)=x^2+8x+t$ will have 2 solns (roots) if the discriminant is positive:

$$-8-4t>0, \text{ that is, } t<-2.$$

It follows that $t=-2$ gives one root and $t>-2$ gives no soln.

#8) This time, the discriminant is t^2+80:

i) $t^2+80>0$ gives 2 solns.

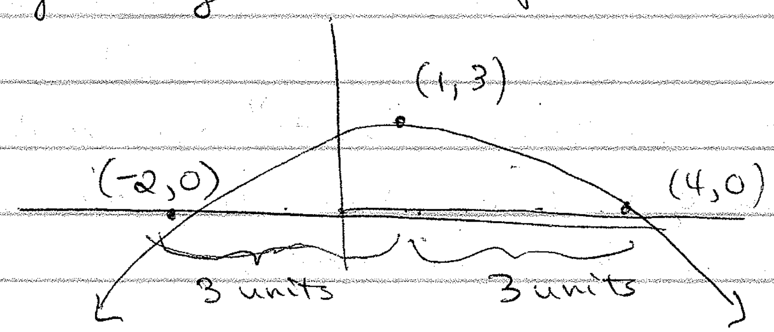
That is, $t^2>-80$, which is true $\underline{\text{for all } t}$.

(ii + iii don't occur since

$t^2+80=0$ has no solution,

nor does $t^2+80<0$.)

#9) Symmetry of the parabola gives the answer:



#3) Be sure the equation is in $ax^2+bx+c=0$ form first.

a) $x^2-6x-7=0$

$$x=\frac{6\pm\sqrt{36-(-28)}}{2}$$

$$=\frac{6\pm\sqrt{64}}{2}=\frac{6\pm 8}{2}$$

$x=7,-1$

b) $x^2-10x=3 \longrightarrow x^2-10x-3=0$

$$x=\frac{10\pm\sqrt{100+12}}{2}=\frac{10\pm\sqrt{112}}{2}=\frac{10\pm 4\sqrt{7}}{2}$$

$16 \times 7 = 112$

$x=5\pm 2\sqrt{7}$

c) $1+3x+x^2=0$

$$x=\frac{-3\pm\sqrt{9-4}}{2}=\frac{-3\pm\sqrt{5}}{2}$$

d) $-3x^2+2x+5=0$ $\quad$ $+3x^2-2x-5=0$

$$x=\frac{-2\pm\sqrt{4-4(-15)}}{-6}$$

$$=\frac{-2\pm\sqrt{64}}{-6}=\frac{-2\pm 8}{-6}$$

$x=-1,-5/3$

e) $2x^2+x+7=0$

$$x=\frac{-1\pm\sqrt{1-4(14)}}{4}=\cancel{\frac{-1\pm\sqrt{55}}{4}}$$

no roots $\Longleftarrow$ $\dfrac{-1\pm\sqrt{-55}}{4}$

For our purposes, "roots" are real numbers. If you get a negative discriminant, the equation has no roots.