Sec. 2.4 HW

#1) a) $\sum_{i=1}^{5} \sqrt{i} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5}$

b) $\sum_{i=0}^{4} \frac{2i-1}{2i+1}$ i is the index, starting at 0 & ending at 4

$$\sum = \frac{2(0)-1}{2(0)+1} + \frac{2(1)-1}{2(1)+1} + \frac{2(2)-1}{2(2)+1} + \frac{2(3)-1}{2(3)+1} + \frac{2(4)-1}{2(4)+1}$$

$$= -1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + \frac{7}{9}$$ (done enough!)

c) $\sum_{i=4}^{6} i^3 = 4^3 + 5^3 + 6^3 = $ etc (book says stop here)

d) $\sum_{i=0}^{n-1} (-1)^i$ This tells us to start at 0 & express the summands up to the $(n-1)$ summand:

$$(-1)^0 + (-1)^1 + (-1)^2 + \cdots + (-1)^{n-2} + (-1)^{n-1}$$

$$\underbrace{1 - 1 + 1 - 1 + \cdots}_{\text{Do not cancel!}} + (-1)^{n-1}$$

which will be -1 if n is even & $+1$ if n is odd

The answer is 0 or 1 depending on n.

e) $\sum_{i=n}^{n+3} i^3$ Just plug in n for i:

$$\sum = n^3 + (n+1)^3 + (n+2)^3 + (n+3)^3$$

last term

f) $\sum_{i=3}^{6} \sqrt[i]{8} = 8^{1/3} + 8^{1/4} + 8^{1/5} + 8^{1/6}$ ok? done!

#2) a) $2+3+4+5+6 = \sum_{i=2}^{6} i$

b) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{19}{20}$

Look for trend:
numerator is 1, 2, 3, ...
denominator is 2, 3, 4...

So, $\frac{i}{i+1}$ would be the summand (that is, the ith term), and Σ notation is:

$$\sum_{i=1}^{19} \frac{i}{i+1}$$

c) $2 - 4 + 6 - 8 + \cdots + -56$

Looks like the terms <u>alternate</u> from (+) to (−) + go up by 2 each time. That is, $\pm 2i$. The way we alternate a series is to multiply each term by $(-1)^{i+1}$, assuming we start at $i=1$ so the first term is (+).

Thus, $\sum_{i=1}^{28} (-1)^{i+1} 2i$ gets us to -56 as the last term (check it:

$(-1)^{28+1} 2(28) = (-1)^{29}(56) = -56$

#2d) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{100}$

Clearly the denom. is i^2, where $i = 1$ is the lower bound of summation. The numer. is always 1. So,

$$\sum_{i=1}^{10} \frac{1}{i^2} = \frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{10^2} = 1 + \frac{1}{4} + \cdots + \frac{1}{100}$$

Answer

#2e) $1 + x + x^2 + x^3 + \cdots + x^n$

Now we're adding powers of x, so the index is on the exponent:

$$\sum_{i=0}^{n} x^i = \underset{\substack{\uparrow \\ 1}}{x^0} + x^1 + x^2 + \cdots + x^n$$

Notice you had to start at $i = 0$ so the first term is 1.

#3) Before showing to solution, I'll explain something. The "lower bound" of the index of summation is just the starting number.

$$\sum_{i=0}^{n} a_i = a_0 + a_1 + \dots + a_n$$

where a_i is the "i^{th} term"

How can I equivalently state

$\sum_{i=0}^{n} a_i$ as $\sum_{i=1}^{n+1} a_?$

In other words, what should the index on "a" look like to compensate for starting 1 higher than 0? It's this:

$$\sum_{i=0}^{n} a_i = \sum_{i=1}^{n+1} a_{i-1}$$

This is given in the book on p. 54 with all kinds of letters ($m, c, i, n \dots$)

Don't let this ~~upset~~ confuse you; just consider how to adjust the i^{th} term

So that the sum <u>doesn't</u> change.

For example:

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + 4 + \cdots + n$$

If we start at i = zero, then the right side changes:

$$\sum_{i=0}^{n-1} = 0 + 1 + 2 + \cdots + n-1$$

But we need it to look the same as $1 + 2 + 3 + \cdots + n$, so we adjust the <u>ith</u> term; since we subtracted 1 on the lower bound, we'll add 1 to the ith term expression:

$$\sum_{i=0}^{n-1} i+1 = (0+1) + (1+1) + (2+1) + \cdots + (n-1+1)$$

$$= 1 + 2 + 3 + \cdots + n$$

$$= \sum_{i=1}^{n} i$$

Notice the upper bound also dropped by 1, from n to $n-1$.

Ex $\displaystyle\sum_{i=1}^{n} 2i+3 \Rightarrow \sum_{i=0}^{n-1} 2(i+1)+3$

$$= \sum_{i=0}^{n-1} 2i+5$$

Check:

$$\sum_{i=1}^{n} 2i+3 = [2(1)+3]+[2(2)+3]+\cdots+[2n+3]$$
$$= 5+7+9+\cdots+(2n+3)$$

$$\sum_{i=0}^{n-1} 2(i+1)+3 = [2(0+1)+3]+[2(1+1)+3]+\cdots+ [2(n\underset{+1}{-1})+3]$$

← careful!

$$= 5+7+\cdots+(2n+3)$$

They're the same.

#3 a) $\displaystyle\sum_{i=1}^{7} (3i^2+2) = \sum_{i=0}^{6} \left[3(i+1)^2 + 2\right]$

add 1

subtract 1

and $\displaystyle\sum_{i=5}^{12} \left[3(i-4)^2 + 2\right]$

subtract 4

add 4

b) $\displaystyle\sum_{i=3}^{5} \frac{i-2}{3i+1} = \sum_{i=0}^{2} \frac{(i+3)-2}{3(i+3)+1}$

add 3

subtract 3

and $\displaystyle\sum_{i=5}^{7} \frac{(i-2)-2}{3(i-2)+1}$

subtract 2

add 2

add 6

c) $\displaystyle\sum_{i=6}^{10} (-4i+10)^2 = \sum_{i=0}^{4} \left[(-4(i+6)+10\right]^2$

subtract 6

and $\displaystyle\sum_{i=5}^{9} \left[-4(i+1)+10\right]^2$

add 1

subtract 1 ~~add~~

4) These use special sums: (p. 56)

- $\sum_{i=1}^{n} c = nc$
- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

And also, the following properties: (p. 55)

- $\sum_{i=m}^{n} a_i$ has $n-m+1$ terms
- $\sum_{i=m}^{n} c = c(n-m+1)$, c is a constant
- $\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$, c is a constant
- $\sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i = \sum_{i=m}^{n} (a_i \pm b_i)$

#4)

a) $\sum_{i=4}^{8} (3i-2) = 3(4)-2 + 3(5)-2 + 3(6)-2 + 3(7)-2 + 3(8)-2$

$= 10 + 13 + 16 + 19 + 22$

$= \boxed{80}$

b) $\sum_{i=0}^{4} 3^i = 3^0 + 3^1 + 3^2 + 3^3 + 3^4 = \boxed{121}$

c) $\sum_{i=0}^{4} (2^i + i^2) = (2^0 + 0^2) + (2^1 + 1^2) + (2^2 + 2^2) + (2^3 + 3^2) + (2^4 + 4^2)$

$= 1 + 3 + 8 + 17 + 32$

$= \boxed{61}$

d) $\sum_{i=0}^{5} \binom{5}{i}$ skip "5 choose i" is the topic

e) $\sum_{i=1}^{10} 7 = 7(10-1+1) = \boxed{70}$ from $\sum_{i=m}^{n} c = c(n-m+1)$. (This is also $\sum_{i=1}^{n} c = nc$)

f) $\sum_{i=1}^{5} i^3 = \left[\frac{5(5+1)}{2}\right]^2 = 15^2 = \boxed{225}$

from $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$