#4 g) $\sum_{i=1}^{15} i = \frac{15(15+1)}{2} = \boxed{120}$

h) $\sum_{i=20}^{75} i = ?$

You ~~[illegible]~~ memorize the formula

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

and note that

$$\sum_{i=1}^{19} i + \sum_{i=20}^{75} i = \sum_{i=1}^{75} i$$

Rearranging:

$$\sum_{i=20}^{75} i = \sum_{i=1}^{75} i - \sum_{i=1}^{19} i$$

$$= \frac{75(76)}{2} - \frac{19(20)}{2}$$

$$= (75)(38) - (19)(10)$$

$$\boxed{= 2660}$$

#4 i) $\sum_{i=1}^{15} 3i^2 = 3\sum_{i=1}^{15} i^2 = 3\left[\frac{15(15+1)(2\cdot 15+1)}{6}\right]$

from the formula for $\sum_{i=1}^{n} i^2$

This $= 3\left(\frac{15\cdot 16\cdot 31}{6}\right)$

$= \frac{15\cdot 16\cdot 31}{2}$

$\boxed{= 3720}$

j) $\sum_{i=1}^{8} (i^2 - i) = \sum_{i=1}^{8} i^2 - \sum_{i=1}^{8} i$

$= \frac{8(8+1)(2\cdot 8+1)}{6} - \frac{8(8+1)}{2}$

$= \frac{8\cdot 9\cdot 17}{6} - \frac{8\cdot 9}{2}$ etc

k) $\sum_{i=1}^{20} (i+1)(i-1) = \sum_{i=1}^{20} i^2 - 1 = \sum_{i=1}^{20} i^2 - \sum_{i=1}^{20} 1$

$= \frac{20(20+1)(2\cdot 20+1)}{6} - 20\cdot 1$

$= \boxed{2850}$

#5) $\sum_{i=1}^{n} i = 78$ What's n?

$$\Downarrow \qquad \Downarrow$$

$$\frac{n(n+1)}{2} = 78$$

$$n^2 + n = 156$$

$$n^2 + n - 156 = 0$$

$$(n-12)(n+13) = 0 \longrightarrow \boxed{n = 12}$$

(discard -13)

#7) a) $\sum_{i=1}^{n} 2i = 2\sum_{i=1}^{n} i = \cancel{2}\boxed{\frac{n(n+1)}{\cancel{2}}}$

b) $\sum_{i=1}^{n} 6 = \boxed{6n}$

c) $\sum_{i=1}^{n} (2-5i) = \sum_{i=1}^{n} 2 - \sum_{i=1}^{n} 5i$

$$= 2n - \frac{5(n)(n+1)}{2}$$

$$= \frac{4n - 5n^2 - 5n}{2} = \boxed{\frac{-5n^2 - n}{2}}$$

#7 a) $\sum_{i=1}^{n} (i+1)(i+2)$

$$= \sum_{i=1}^{n} i^2 + 2i + i + 2$$

$$= \sum_{i=1}^{n} i^2 + 3i + 2$$

$$= \sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 2$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + 2n$$

Expand + get LCD = 6

Combine like terms + reduce

to get final

$$\frac{n^3 + 6n^2 + 11n}{3}$$

7e & f, g etc —

These entail using the formulas & doing some algebra. Also, noticing the lower bound on the sum is different for ~~two~~ problems.

7e) looks like # 7d) but $i = 0$.

It still sums to n (that is, we aren't writing an equivalent sum to $n-1$)

So, there's one more term than $\sum_{i=1}^{n}$

Thus, $\sum_{i=0}^{n} (i+1)(i+2) = (0+1)(0+2) + \sum_{i=1}^{n} (i+1)(i+2)$

That is, $\boxed{\sum = 2 + \text{answer to } \# 7e!}$

We'll cover these & the last bit (~~Dede~~ (telescoping sums) on Monday.