Math 108 - Selected Ch. 1 solns.

Sec 1.1 # 2e $\frac{3}{5} = 3 \div 5 = 5\overline{)3.0}^{\,.6} = .6$

#3: These are all rational numbers because you can write them as fractions (i.e., terminating or repeating decimals)

a) $12 = 12/1$

b) $-.36 = \frac{-36}{100}$

c) $21.7 = 217/10$

d) $-.\overline{5} = ?/?$

$$\text{Let } x = .555\ldots$$
$$\text{So } 10x = 5.555\ldots$$
$$\left.\right\} \text{subtract}$$
$$9x = 5 \longrightarrow x = 5/9$$

e) $.\overline{27} = .2727\ldots = x$

$$27.2727\ldots = 100x \quad \left.\right\} \text{subtract}$$
$$27 = 99x \longrightarrow x = 27/99$$

#7. a)

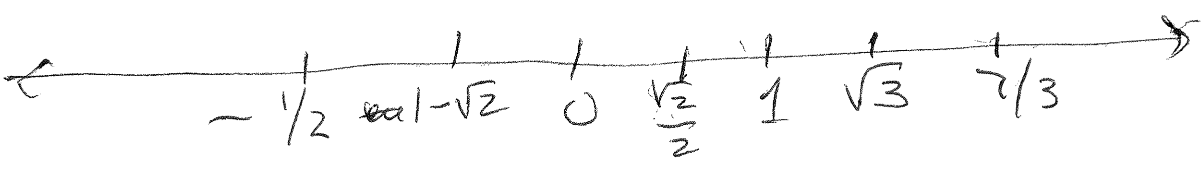
$-1/2$	rational
0	rational, integer
-2π	irrational
$\sqrt{3}$	irrational
1	rational integer
$\sqrt{2}/2$	irrational
$7/3$	rational
$1-\sqrt{2}$	irrational

Know:

$\sqrt{2} \approx 1.414$

$\pi \approx 3.1415$

$\sqrt{3} \approx 1.732$



Sec 1.2 Selected solns.

1. $32^{4/5} = (32^{1/5})^4 = 2^4 = 16$;

$17^0 = 1$; $8^{-1/3} = \frac{1}{8^{1/3}} = \frac{1}{2}$;

$4^{3/2} = (4^{1/2})^3 = 2^3 = 8$;

$100^{1/2} - 64^{1/2} = 10 - 8 = 2$; $(100-64)^{1/2} = 36^{1/2}$
$= 6$

2. b) $\frac{5x^{-2}yz^3}{(2x)^{-2}(yz^3)^2} = \frac{5yz^3 \cdot (2x)^2}{x^2 (yz^3)^2}$

$= \frac{5 \cdot y \cdot z^3 \cdot 4 \cdot \cancel{x^2}}{\cancel{x^2} \cdot y^2 \cdot z^6} = \frac{20}{yz^3}$

d) $\left(\frac{xy^5}{3x^{-2}yz}\right)^{-2} = \left(\frac{x x^2 y^5}{3}\right)^{-2}$

$= \left(\frac{x^3y^5}{3}\right)^{-2} = \left(\frac{3}{x^3y^5}\right)^2 = \frac{9}{x^6y^{10}}$

3. $\sqrt[3]{x^5} = x^{5/3}$, $\frac{x}{\sqrt[5]{x^3}} = \frac{x}{x^{3/5}} = x^{1-3/5}$
$= x^{2/5}$

4. $-x^{1/2} = -\sqrt{x}$, $x^{9/5} = \sqrt[5]{x^9}$

Must avoid zero in denom, negative under a radical, where the radical is an even root.

5. $\sqrt{x}$ is defined for $x \geq 0$

$\sqrt{-x}$ is defined for $-x \geq 0$, so $x \leq 0$

$\sqrt{x^2}$ is defined for all $x \in \mathbb{R}$

$\frac{1}{\sqrt{x}}$ is defined for $x > 0$

$\sqrt{x-6}$ for $x - 6 \geq 0$, so $x \leq 6$

$\sqrt[3]{x}$ for all x since positive, negative, and zero all have odd roots. Only even roots need non-negative radicand.

8. a) $$\frac{3}{\sqrt[3]{5}} = \frac{3}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5}\,\sqrt[3]{5}}{\sqrt[3]{5}\,\sqrt[3]{5}} = \frac{3\sqrt[3]{25}}{5}$$

b) $$\frac{1}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{9}\,\sqrt[3]{9}}{\sqrt[3]{9}\,\sqrt[3]{9}} = \frac{\sqrt[3]{81}}{9} = \frac{\sqrt[3]{27}\,\sqrt[3]{3}}{9}$$

$$= \frac{3\sqrt[3]{3}}{9} = \sqrt[3]{3}/3$$

c) $$\frac{\sqrt{7}}{\sqrt[3]{49}} \cdot \frac{\sqrt[3]{49}\,\sqrt[3]{49}}{\sqrt[3]{49}\,\sqrt[3]{49}} = \frac{\sqrt{7}\,\sqrt[3]{7^2 \cdot 7^2}}{49}$$

$$= \frac{\sqrt{7}\,\sqrt[3]{7^4}}{49} = \frac{\sqrt{7}\;7\sqrt[3]{7}}{49} = \frac{\sqrt{7}\,\sqrt[3]{7}}{7}$$

8d) $\frac{6}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{6\sqrt{5}-6\sqrt{2}}{5-2} = \frac{6\sqrt{5}-6\sqrt{2}}{3}$

$= 2(\sqrt{5}-\sqrt{2})$

e) $\frac{\sqrt{3}}{\sqrt{3}-\sqrt{7}} \cdot \frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}+\sqrt{7}} = \cancel{\frac{3+3\sqrt{7}}{3-7} = \frac{3+3\sqrt{7}}{-4}}$

$= \frac{3+\sqrt{21}}{3-7} = \frac{3+\sqrt{21}}{-4}$

f) $\frac{\sqrt{2}}{\sqrt{6}-2} \cdot \frac{\sqrt{6}+2}{\sqrt{6}+2} = \frac{\sqrt{12}+2\sqrt{2}}{6-4}$

$= \frac{2\sqrt{3}+2\sqrt{2}}{2} = \sqrt{3}+\sqrt{2}$

9\. a) $\sqrt{8}\sqrt{2} = \sqrt{16} = 4$

b) $\sqrt[3]{6}\sqrt[3]{9} = \sqrt[3]{54}$

$= \sqrt[3]{27}\cdot\sqrt[3]{2}$

$= 3\sqrt[3]{2}$

c) $\cancel{\frac{2}{\sqrt[3]{16}}}$

d) $\sqrt{8}+\sqrt{2} = 2\sqrt{2}+\sqrt{2} = 3\sqrt{2}$

e) $\sqrt{18}-\sqrt{14}+\sqrt{32} = \sqrt{2\cdot 9}-\sqrt{2\cdot 7}+\sqrt{2\cdot 16}$

$= 3\sqrt{2}-\sqrt{2\cdot 7}+4\sqrt{2} = 6\sqrt{2}-\sqrt{14}$

f) $\sqrt{75}+\sqrt{48}+\sqrt{12}-\sqrt{50} = \sqrt{3\cdot 25}+\sqrt{3\cdot 16}$
$+\sqrt{3\cdot 4}-\sqrt{2\cdot 25}$

$= 5\sqrt{3}+4\sqrt{3}+2\sqrt{3}-5\sqrt{2} = 11\sqrt{3}-5\sqrt{2}$

g) $\left(\frac{\sqrt{3}-\sqrt{6}}{2}\right)^2 = \left(\frac{\sqrt{3}-\sqrt{6}}{2}\right)\left(\frac{\sqrt{3}-\sqrt{6}}{2}\right)$

$= \frac{3 - \sqrt{3}\sqrt{6} - \sqrt{3}\sqrt{6} + 6}{4}$

$= \frac{9 - 2\sqrt{3}\sqrt{6}}{4} = \frac{9 - 2\sqrt{18}}{4}$ $\cancel{= 9(}$

$= \frac{9 - 2\sqrt{9 \cdot 2}}{4} = \frac{9 - 2 \cdot 3\sqrt{2}}{4}$

$= \frac{9 - 6\sqrt{2}}{4}$

Sec 1.3 1d) $3x^2 - [4x - x(2x+1)]$ $\cancel{= 6x^2 - 4x}$

$= 3x^2 - [4x - 2x^2 - x]$

$= 3x^2 - 4x + 2x^2 + x = 5x^2 - 3x$

2) e) $(x+2)^2 = (x+2)(x+2) = x^2 + 2x + 4$

d) $(2x+3)(4x^3 - x - 1) = 8x^4 - 2x^2 - 2x$

$+ 12x^3 - 3x - 3$

$= 8x^4 + 12x^3 - 5x - 3$

g) $2x^{1/3}(3x^{2/3} + x^3)$

$= 2x^{1/3} \cdot 3x^{2/3} + 2x^{1/3} \cdot x^3$

$= 2x^{1/3+2/3} + 2x^{1/3+3} = 2x^1 + 2x^{7/3}$

Sec 1.4

#1) $(4,444,444 + 4,444,443)(4,444,444 - 4,444,443)$

$= (8,888,887)(1) = 8,888,887$

#2) a) $x^2 - x - 12 = (x-4)(x+3)$

b) $x^2 - 13x + 22 = (x-11)(x-2)$

c) $x^2 + 2x - 35 = (x-5)(x+7)$

d) $3x^2 - 18x + 15 = 3(x^2 - 6x + 5)$

$= 3(x-5)(x-1)$

e) $4x^2 + 40x + 100 = 4(x^2 + 10x + 25)$

$= 4(x+5)(x+5) = 4(x+5)^2$

f) $2x^2 - x^{-1} + x^{-4} = x^{-4}\left(\frac{2x^2}{x^{-4}} - \frac{x^{-1}}{x^{-4}} + \frac{x^{-4}}{x^{-4}}\right)$

$= x^{-4}(2x^6 - x^3 + 1)$

(we factored out smallest power of x)

g) $(x^3 - 2x^2) + (4x - 8) = x^2(x-2) + 4(x-2)$

$= (x^2+4)(x-2)$

h) $3x^2 + 5x + 2 = (3x+2)(x+1)$ [illegible]

First guess was right.

i) $3x^3 + 24 = 3(x^3+8) = 3(x+2)(x^2-2x+4)$

j) $-2x^2 + 6x + 8 = -2(x^2 - 3x - 4)$

$= -2(x-4)(x+1)$

k) $1 - x^{16} = (1+x^8)(1-x^8)$

$= \underbrace{(1+x)(1+x^2+x^4)}_{\text{sum of cubes factored}}\underbrace{(1-x)(1+x^2+x^4)}_{\text{difference of cubes factored}}$

l) $(x^5 + 2x^3) + (x^2+2) = x^3(x^2+2) + (x^2+2)$

$= (x^3+1)(x^2+2) \longrightarrow$

2l) $= (x+1)(x^2-x+1)(x^2+2$

2m) $6x(2x+1)^{-1/2} + 3x^2(2x+1)^{\frac{1}{2}}$

$= 3x(2x+1)^{-1/2}\left[2 + x(2x+1)\right]$

2n) $x^4+4x^2-5 = (x^2-1)(x^2+5)$

$= (x-1)(x+1)(x^2+5)$

2o) $2x^5 - 16x^4 + 32x^3 = 2x^3(x^2-8x+16)$

$= 2x^3(x-4)(x-4)$

2p) $12x^2+16x-3$ Guess (hard) or magic factor (better)

$(12)(-3) = -36$: factors $(-12)(3)$, $(2)(-18)$

→ Combines to give 16, so use it.

$(12x^2-18x)+(2x-3)$

$= 6x(2x-3)+(2x-3) = (2x-3)(6x+1)$

2q) $-27x^3+1 = 1-27x^3$ Diff of cubes

$= (1-3x)(1+3x+9x^2)$