HW – Ch. 10.2 continued:

#4) $\log_9 x = \frac{1}{2} \rightarrow x = 9^{1/2} = \sqrt{9} = \boxed{3}$ (positive only)

$\log_x 11 = 1 \rightarrow x^1 = 11 \rightarrow \boxed{x = 11}$

$\log_x 27 = \frac{3}{2} \rightarrow 27 = x^{3/2} \rightarrow (27)^{2/3} = x$

$\rightarrow (27^{1/3})^2 = 3^2 = \boxed{9 = x}$

$\log_6 x = -2 \rightarrow x = 6^{-2} = 1/6^2 = \boxed{1/36}$

#5 $\log_6 37 \underset{?}{\gtrless} \log_7 48$

Known: $\log_6(36+1) \neq \log_6 36 + \log_6 1$
wrong approach!!

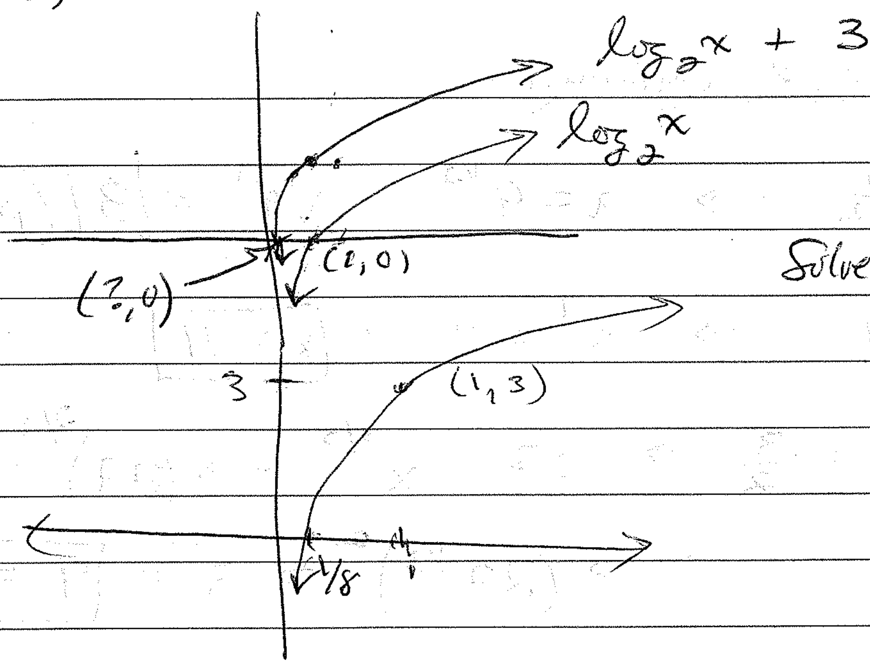
But $\log_6 37 > \log_6 36 = 2$ (see graph of $\log_6 x$)

and $\log_7 48 < \log_7 49 = 2$ (see graph of $\log_7 x$)

So $\log_6 37 > 2$ + $\log_7 48 < 2$

Hence $\log_6 37 > \log_7 48$ ✓

#6) a) $f(x) = \log_2 x + 3$



VA: $x = 0$

What is x-int?

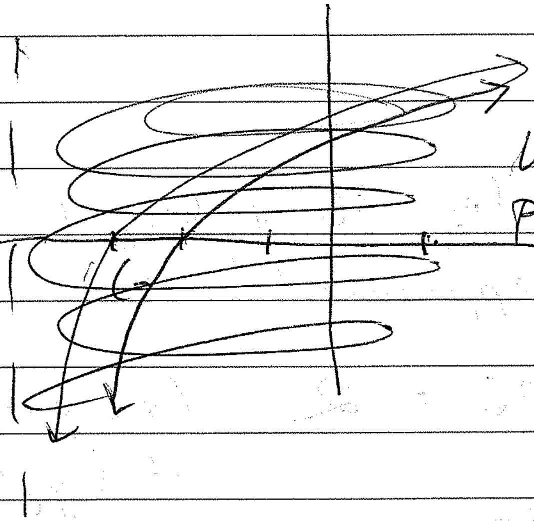
Solve $\log_2 x + 3 = 0$

$\log_2 x = -3$

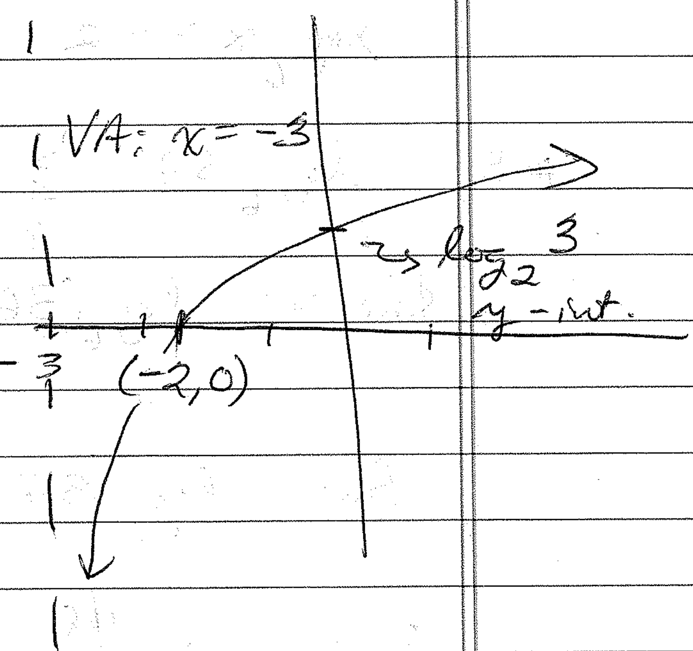
$x = 2^{-3} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$

root

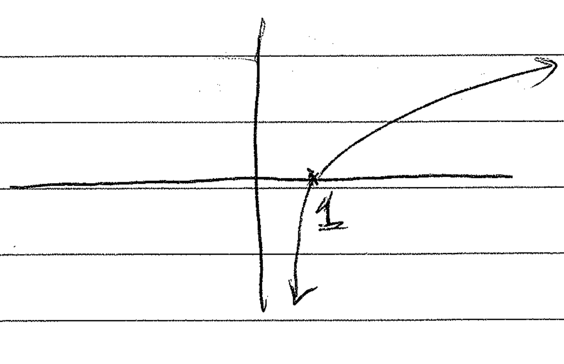
b) $f(x) = \log_2(x+3)$



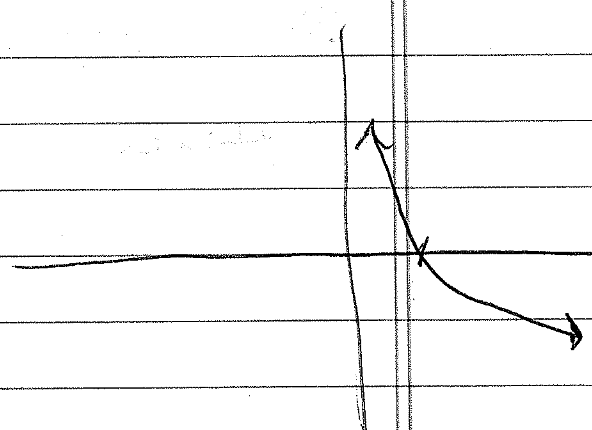
VA: $x = -3$



c) $f(x) = -\ln x = -\log_e x$

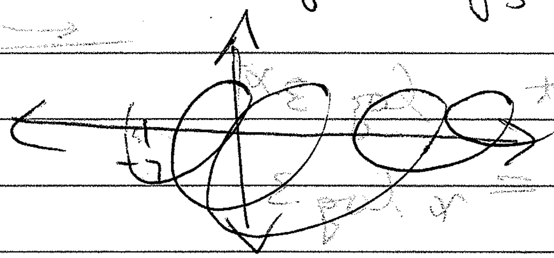


reflect $\longrightarrow$ over x-axis



#7) a) $f(x) = \log_3(x-7) - \log_3(x+2)$

The domain is the intersection of the doms. of $\log_3(x-7)$ & $\log_3(x+2)$



Use pencil!

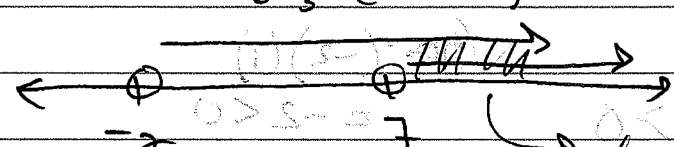
You could note that $\log_3(x-7) - \log_3(x+2)$ $= \log_3 \frac{x-7}{x+2}$ & its domain is that for which $\frac{x-7}{x+2} > 0$, which is clearly $\boxed{x > 7}$. But also, $x \neq -2$. No problem there, since $-2 < 7$.

So, $\log_b a = y$ has domain $a > 0$ is one way to find dom.

Also, $\log_b a \pm \log_b c$ has dom of the more restrictive of $a > 0$ or $b > 0$

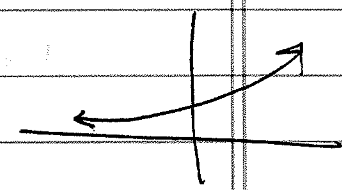
Since dom $\log_3(x-7)$ is $x > 7$
& dom $\log_3(x+2)$ is $x > -2$

then



dom $= (7, \infty)$

#7 b) $f(x) = \log 3^x$. Antilog 3^x must be > 0.
The exponent can be anything, since
$3^x > 0$ for all x.



~~Also~~ Also, note that $\log 3^x$
$= x \log 3$

Since $\log 3$ is a constant, $f(x) = (\log 3)x$
is a line w/ slope $\log 3$ through $(0,0)$

$\boxed{x \in \mathbb{R}}$

c) $f(x) = \ln(e - x)$ Just solve $e - x > 0$
$\longrightarrow \boxed{x < e}$

d) $f(x) = \ln(x^2 - x - 2)$ Solve $x^2 - x - 2 > 0$
by checking intervals
around roots

$x^2 - x - 2 = (x-2)(x+1) = 0 \rightarrow x = 2, -1$

-1 $\qquad$ 2

check $x = -2$ $\qquad$ check $x = 0$ $\qquad$ check $x = 3$

$(x-2)(x+1)$
$= (-4)(-1) = 4 > 0$
yes

$(-2)(1)$
$= -2 < 0$
no

$(1)(4) = 4 > 0$
yes

dom: $(-\infty, -1)$
$\cup (2, \infty)$

#8) $f(x) = \log_3(x+2)$

Dom: $x+2>0 \longrightarrow \boxed{x>-2 \text{ Dom}}$

$f^{-1}(x) = ?$ By def $y = \log_3(x+2)$ is same as writing

$$3^y = x+2$$

The inverse is formed by reversing x, y:

$$3^x = y+2$$

$$y = \boxed{3^x - 2 = f^{-1}(x)}$$

Range of f^{-1} = Dom of f

so $\boxed{y > -2 \text{ is Range } f^{-1}}$

#9) a) $\log_3 x + \log_3 2 = \log_3(2x)$

b) $\log_2 9 - \log_2 y = \log_2(9/y)$

c) $2\log x - 5\log y = \log x^2 - \log y^5$

$= \log(x^2/y^5)$

d) $\frac{1}{2}\log_4(x+5) = \log_4(x+5)^{1/2} = \log_4\sqrt{x+5}$

e) $-4\log_6(2x) = \log_6(2x)^{-4} = \log_6\left(\frac{1}{2x}\right)^4$

$= \log_6(1/16x^4)$

f) $3\ln x + 4\ln y - 4\ln z =$

$\ln x^3 + \ln y^4 - \ln z^4 = \boxed{\ln \dfrac{x^3 y^4}{z^4}}$

g)

~~#10) "[illegible]"~~

#10) a) $\log_3 (y/2) = \log_3 y - \log_3 2$

b) $\log(10x) = \log_{10} 10x = \log 10 + \log x = 1 + \log x$

c) $\log_6 (1/x^3) = \log_6 1 - \log_6 x^3 = 0 - 3\log_6 x = -\log_6 x$

d) $\log_4 (4x^2 y) = \log_4 4 + \log_4 x^2 + \log_4 y$

$= 1 + 2\log_4 x + \log_4 y$

e) $\log_4 (4xy)^2 = 2\log_4 4xy$

$= 2(\log_4 4 + \log_4 x + \log_4 y)$

$= 2(1 + \log_4 x + \log_4 y)$

f) $\log\left(\dfrac{x^2-1}{x^3}\right) = \log(x^2-1) - \log x^3$

$= \log[(x+1)(x-1)] - 3\log x$

$= \log(x+1) + \log(x-1) - 3\log x$

g) $\log \sqrt[5]{x^2/y^3} = \frac{1}{5}\log(x^2/y^3) = \frac{2}{5}\log x - \frac{3}{5}\log y$

h) $\log_2 \sqrt{x}/x^4 = \frac{1}{2}\log_2 x - 4\log_2 x$