Sec 30 Antiderivatives (aka indefinite integrals)

Suppose $f(x) = 1$. What fcn, when differentiated would result in $f(x) = 1$? There are many, in fact, an infinite family of fcns $F(x) + C$ will have derivative $f(x) = 1$.

For example: x, $x+2$, $x-19$, $x+\pi$ and so on, are all antiderivatives of $f(x)$.

So $F(x) = \boxed{x \text{ is an antiderivative of } 1}$
$f(x) = 1$ since $F'(x) = 1$.

Also, $(F(x) + C)' = F'(x) + C' = F'(x)$

E.g.
$$(x+2)' = 1 + 0 = 1$$
$$(x-19)' = 1 + 0 = 1$$
$$(x+\pi)' = 1 + 0 = 1$$

You get the idea.

Def An antiderivative of a fcn $f(x)$ is any fcn $F(x)$ where $F'(x) = f(x)$.

The family of fcns. $F(x) + C$ comprise all antiderivatives of $f(x)$, since
$$F'(x) + C' = f(x) + 0 = f(x)$$

Another name for "antiderivative" ~~is~~ of $f(x)$
"indefinite integral" of $f(x)$

The process of finding ~~one~~ an antiderivative is called "integration" or "antidifferentiation"

The notation for the relationship $F'(x) = f(x)$ (or $[F(x) + C]' = f(x)$) is:

$$\boxed{\int f(x)\,dx = F(x) + C}$$

Constant of integration

Aside Same as

The "dx" is as before the "differential of x". Its physical meaning will become clear when we do "definite integration" of f(x)

We say that we "differentiate with respect to x"

we'll "integrate with respect to x"

Like differentiation, integration has properties and some basic formulas/rules:

1. $\int 1\,dx$ (or just $\int dx$) $= x + C$

2. $\int k \cdot 1\,dx = k \int 1\,dx = kx + C$

3. $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

4. $\int k f(x)\,dx = k \int f(x)\,dx$

Power Rule 5. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$, $\boxed{n \neq -1}$

6. $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \int \dfrac{dx}{x} = \boxed{\ln|x| + C}$

Exp Rule 7. $\int e^x\,dx = e^x + C$

Suppose we'd like to narrow down our answer to just one fcn, not all $F(x)+C$. We do so via

Initial conditions (aka boundary conditions)

Consider the fcns. of displacement, $s(t)$, $v(t)$, $a(t)$. If we consider constant acceleration "a" then the fcns. relate as follows:

$$s(t) = \frac{at^2}{2} + v_0 t + s_0 \qquad \text{position (distance)}$$

$$s'(t) = v(t) = at + v_0 \qquad \text{velocity}$$

$$s''(t) = v'(t) = a(t) = a \qquad \text{acceleration}$$

Initial position & initial velocity are $s(0)$ & $v(0)$.

$$s(0) = \frac{a}{2}\cdot 0^2 + v_0 \cdot 0 + s_0 = s_0$$

Notationally, $s(0) \equiv s_0$

Also, $v(0) = a \cdot 0 + v_0 = v_0$

$$v(0) \equiv v_0$$

Now look at how integration relates these fcns:

Since $s'(t) = v(t)$, $\quad s(t) = \int v(t)\,dt$

and $v'(t) = a(t)$, $\quad v(t) = \int a(t)\,dt$