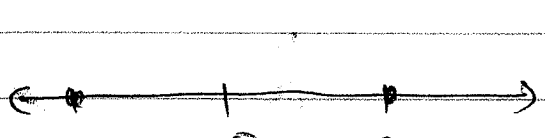
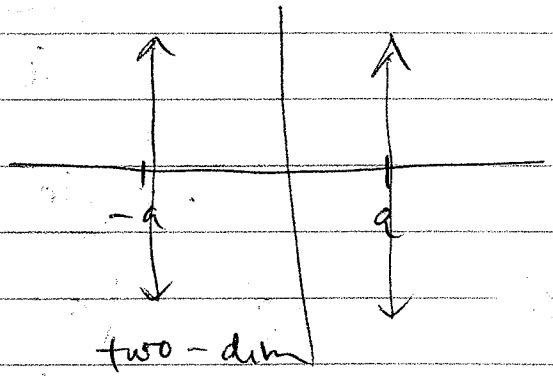
Properties of Absolute Value

① Def $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$

② $|x| = a \rightarrow x = a$ or $x = -a$

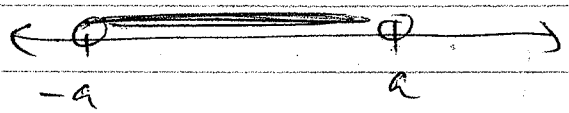


one-dim

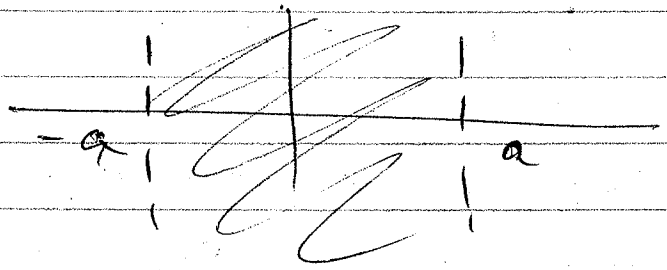


two-dim

③ $|x| < a \rightarrow -a < x < a$



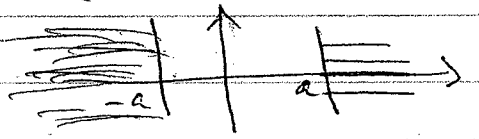
one dimension



two dimensions

$|x| \geq a \rightarrow x \leq -a$ or $x \geq a$

one dim



two dim

(4) $|a||b| = |ab|$

(5) $\frac{|a|}{|b|} = \left|\frac{a}{b}\right| \quad b \neq 0$

(6) $|a|^2 = |a^2| = a^2$

Look at actual numbers to convince yourself (not proofs)

(7) $|a| \overset{\text{def}}{\equiv} \sqrt{a^2} = \sqrt{a}^2$

$|a| = a$, $\sqrt{a^2} = a$, $\sqrt{a}^2 = a$

Triangle Inequalities

(8) $|a| + |b| \geq |a+b|$

e.g. $a=-1, b=2$

$|-1| + |2| = 1+2 = 3 \overset{?}{\geq} |-1+2| = 1$. ✓

(9) $|a| - |b| \leq |a-b|$

e.g. $a=-1, b=2$

$|-1| - |2| = 1-2 = -1 \overset{?}{\leq} |-1-2| = 3$ ✓

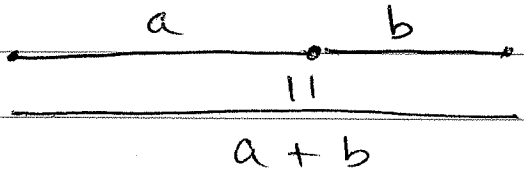
(8) The sum of the lengths of any 2 sides of a triangle must be greater than the third side.

2, 1, 5, 1

$2+5 > 1$ ✓

$2+1 \not> 5$

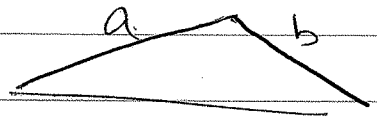
lengths of sticks → $\triangle$?

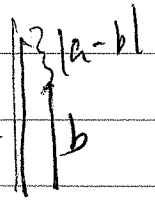


Degenerate triangle (collapsed) but we include in the triangle inequality

$$|a| + |b| \geq |a+b|$$

(9)



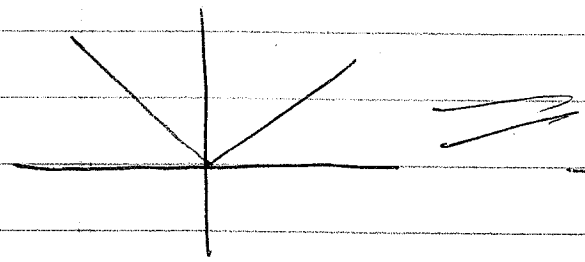


The difference of the lengths of any two sides of a triangle cannot be greater than the third side

Absolute value - notes on equations + inequalities

Sketches

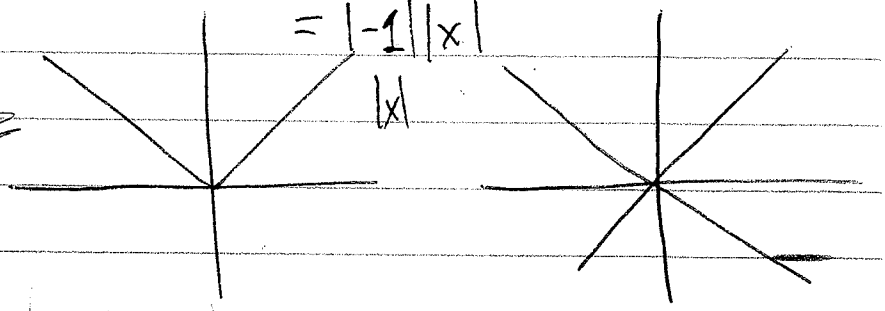
$y = |x|$



$y = |-x| = |-1x|$

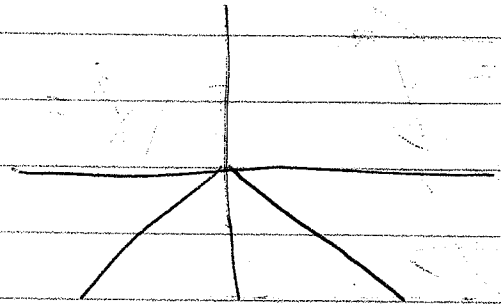
$= |-1||x|$

$|x|$

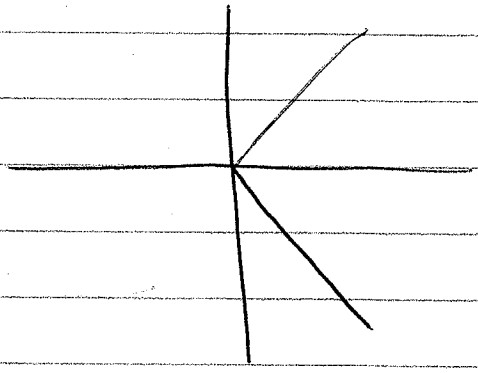


$|y| = |x|$

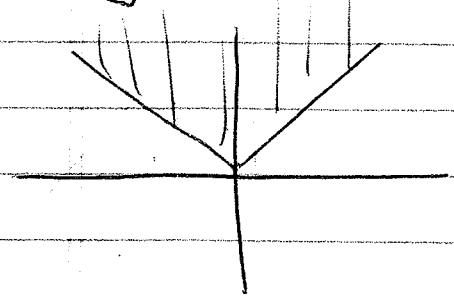
$y = -|x|$



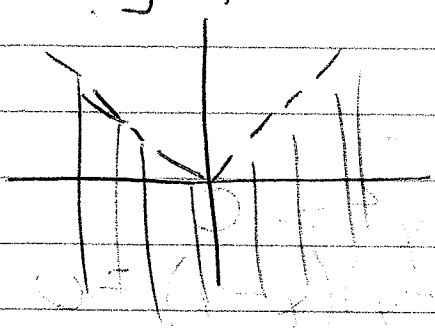
$x = |y|$



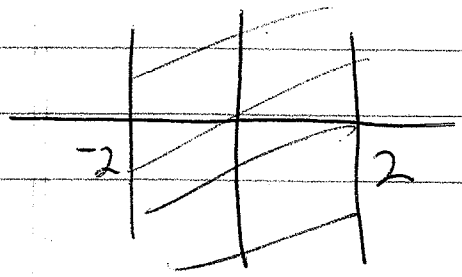
$y \geq |x|$



$y < |x|$



$|x| \leq 2$



$|x| > 2$