Nov 30 Zoom remote lecture

Note! There are 5 more lectures inc. today

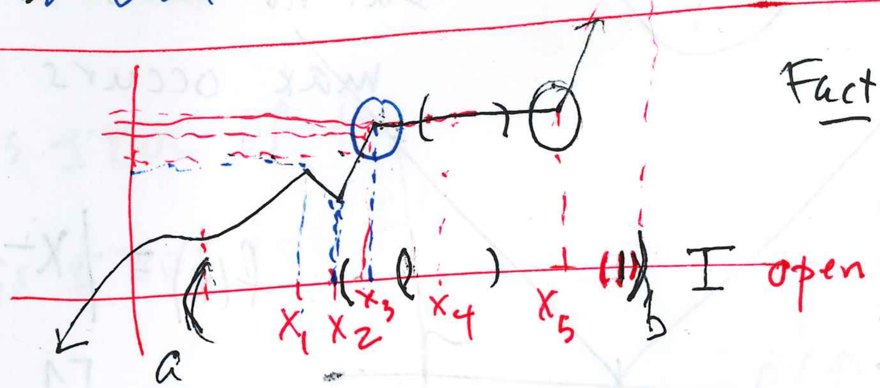
One more in-class quiz (Wed Dec 7) - curve sketching

One more HW to collect (Fri Dec 2)

* Final exam - Mon Dec 12 3:15-5:15 LH 1 *

Homework to hand in ~~Friday~~ Monday: Sec 2.8 #18, Sec 2.7 #20

and ~~Sec 3.4~~ Sec 3.3 #42, #50



Fact Local extremes cannot occur at endpts of $[a,b]$, but absolute extremes can!

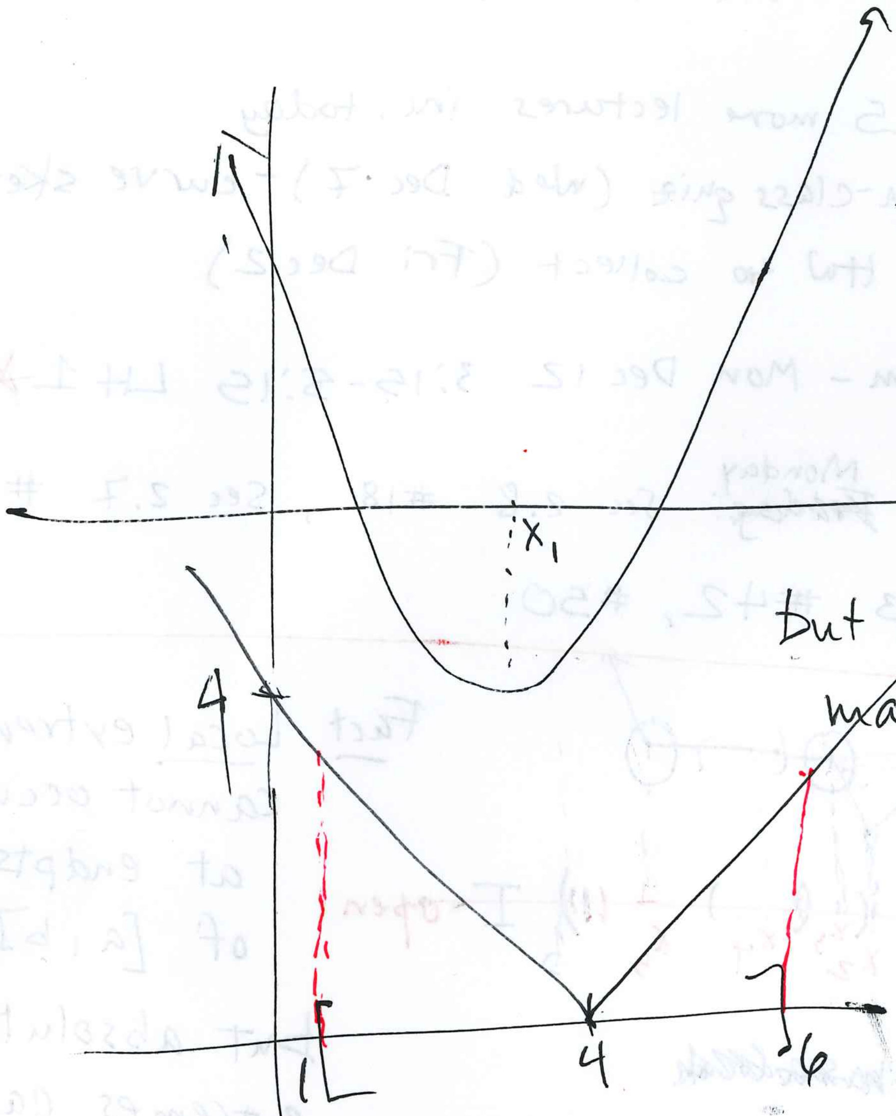
$f(x_3)$ is a local max

$f(x_1)$ is local max

$f(x_2)$ is local min

because $f(x) \le f(x_3)$ for all $x \in \varepsilon$-hood of x_3.

$f(x_4)$ is both loc min/max b/c all $x \in \varepsilon$-hood of x_4 give $f(x) \le f(x_4)$ & $f(x) \ge f(x_4)$



On $\mathbb{R}$, $f(x_1)$ is local + global minimum

But no local or global max occurs

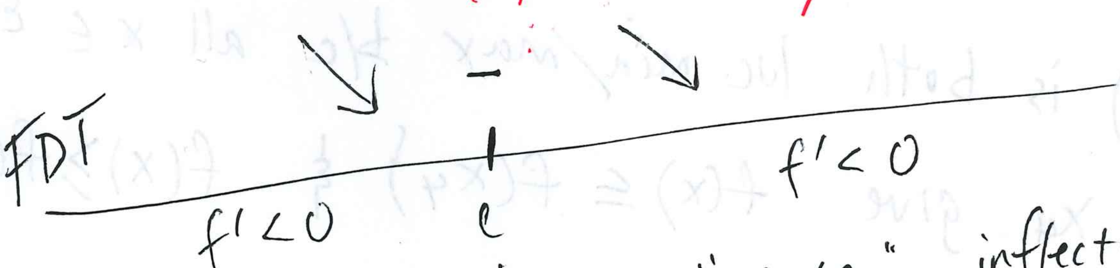
$f(x) = |x-4|$ on $[1, 6]$

$f(4)=0$ loc/abs min

$f(1)=3$ abs max

$f(6)=2$ nothing

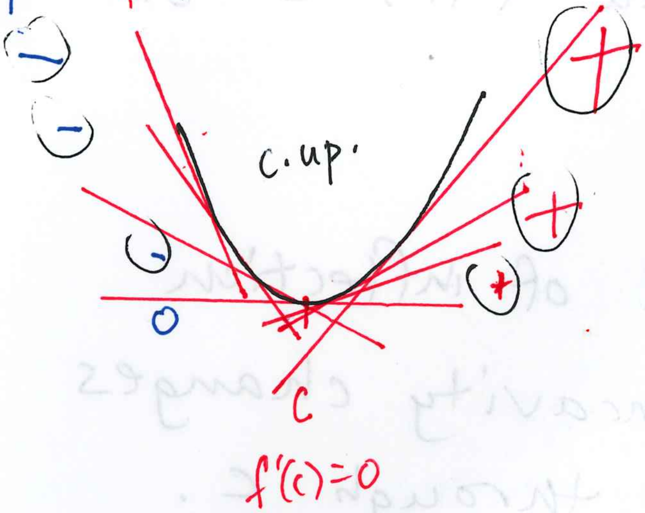
FDT

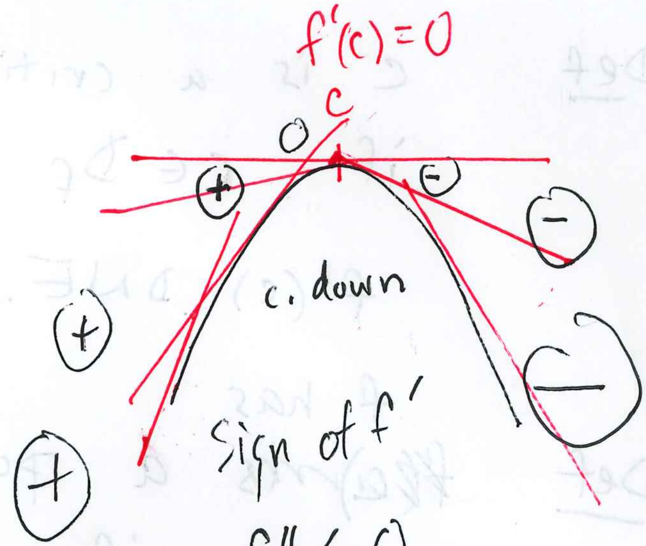


$f'(c)=0$ "pause", inflection pt.

$f' \uparrow$

sign of f', $f'' > 0$





$f'' < 0$

$f' \downarrow$

f' is IROC of f

f'' is IROC of f'

$f \uparrow$, $f' > 0$

$f \downarrow$, $f' < 0$

$f' \uparrow$, $f'' > 0$

$f' \downarrow$, $f'' < 0$

Ex1 $f' < 0$ $f' > 0$

$f \downarrow$ c $f \uparrow$

$f'(c) = 0$

By FDT, $f(c)$ is local min

Ex2

$f \uparrow$ c $f \downarrow$

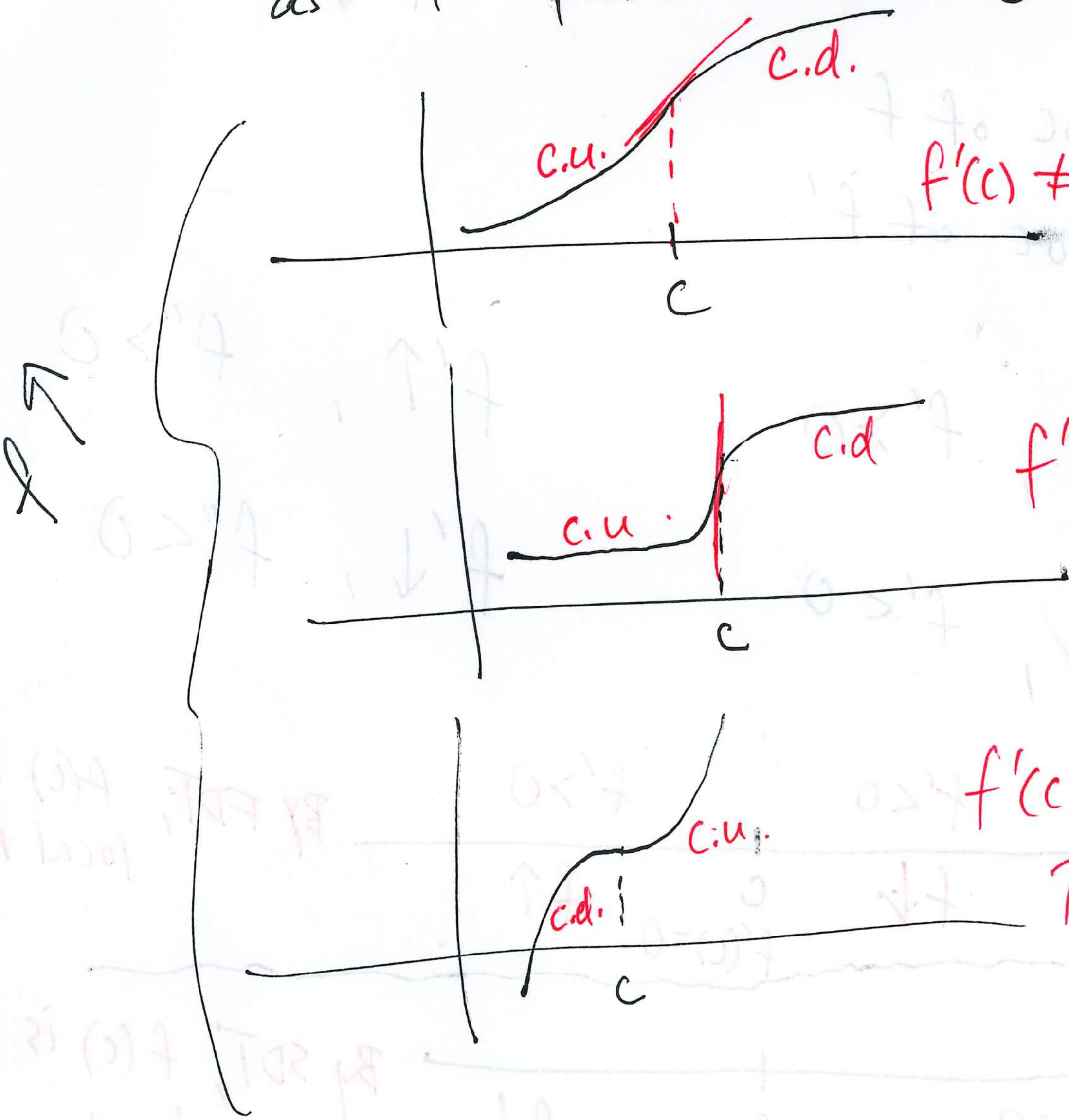
$f' > 0$ $f'(c) = 0$ $f' < 0$

$f''(c) < 0$, c. down

By SDT, $f(c)$ is is local max b/c it lives in c. down interval

Def c is a critical number of $f(x)$ if $c \in D_f$ and $f'(c) = 0$ or $f'(c)$ DNE.

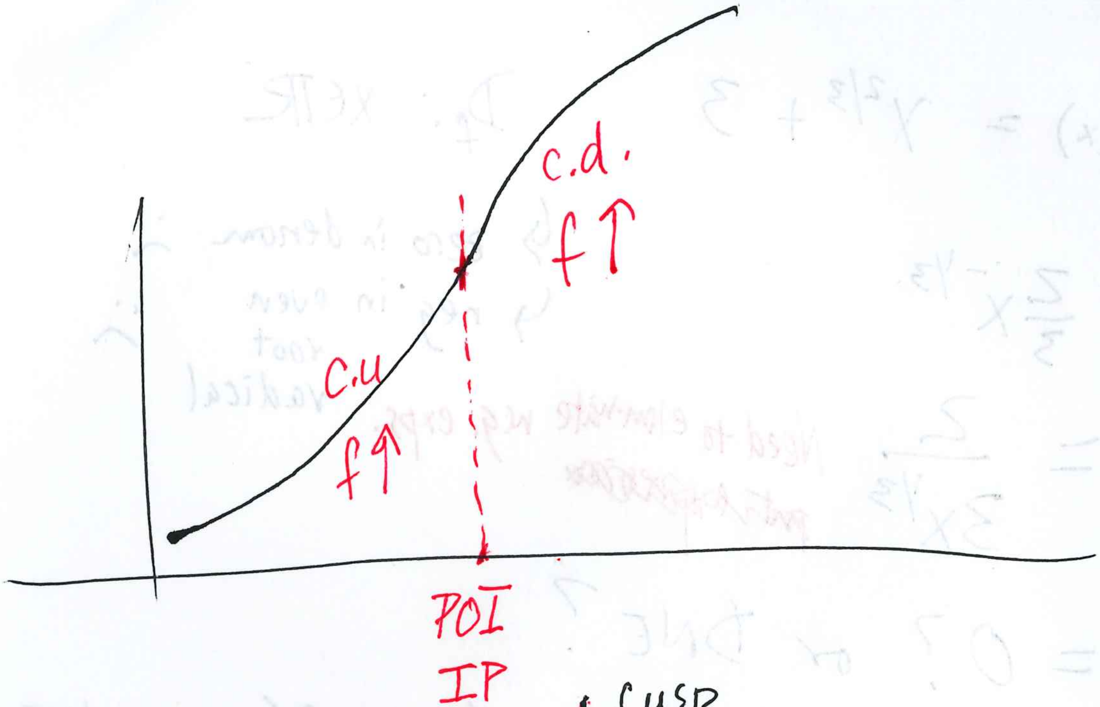
Def f has ~~f(c) is~~ a point of inflection at c if concavity changes as f passes through c.

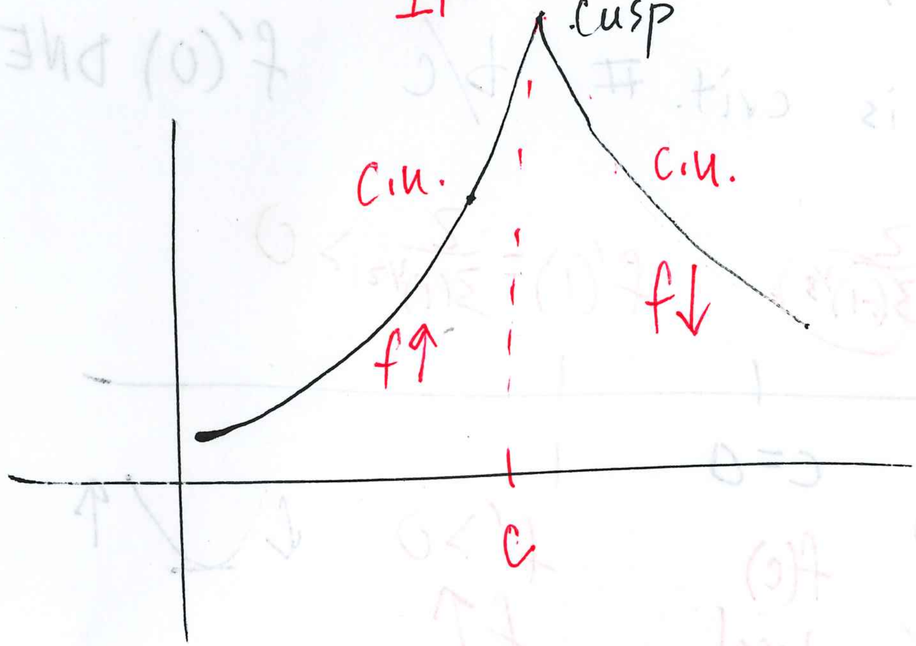


$f'(c) \neq 0$, $\neq$ DNE

$f'(c)$ DNE
vert. tang.

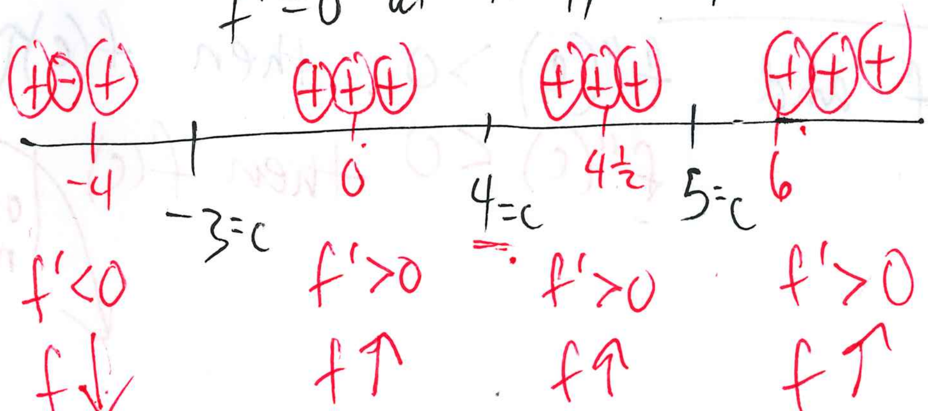
$f'(c) = 0$
pause





#25 $f'(x) = (x-4)^2(x+3)^7(x-5)^8$ — deg 17 $f'(x)$, deg 18 $f(x)$

$f'=0$ at $x=4, -3, 5$



Do sign analysis of f' on intervals formed by c #s.

$\underline{Ex}$ $\quad f(x) = x^{2/3} + 3 \qquad D_f: x \in \mathbb{R}$

↳ zero in denom ☹

↳ neg in even root radical ☹

$f'(x) = \frac{2}{3} x^{-1/3}$

$f' = \frac{2}{3x^{1/3}}$ Need to eliminate neg. exps. ~~put expression~~

$\frac{A}{B} = 0$

$f' = 0$? or DNE?

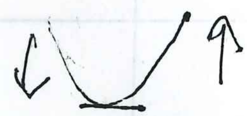
$c = 0$ is crit. # b/c $f'(0)$ DNE

$f'(-1) = \frac{2}{3(-1)^{1/3}}$ $\quad f'(1) = \frac{2}{3(1)^{1/2}} > 0$

FDT

Sign analysis of f'

-1	$c = 0$	1
$f' < 0$	$f(c)$	$f' > 0$
$f \downarrow$	local min	$f \uparrow$



$\underline{\text{Second Derivative Test}}$ - If c is a crit # of f and $f''(c) > 0$ then $f(c)$ local min

$f''(c) < 0$ then $f(c)$

~~$f''(x) =$~~

$$f'(x) = \frac{2}{3x^{1/3}} = \frac{2x^{-1/3}}{3}, \quad f'' = \frac{-2x^{-4/3}}{9} = \frac{-2}{9x^{4/3}}$$

$$f''(x) = \frac{-2}{9x^{4/3}}$$

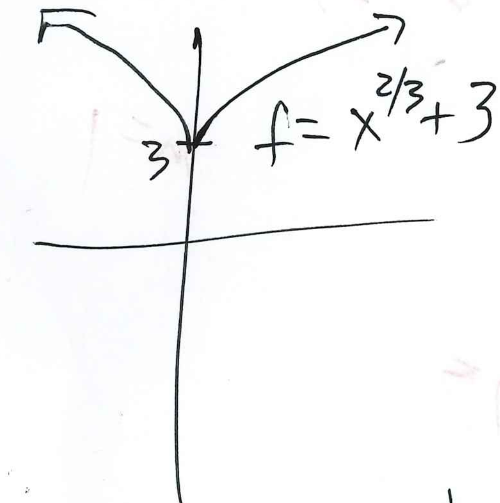
$\cancel{\frac{(-)}{(+)}}$ $\quad$ $\frac{(-)}{(+)}$

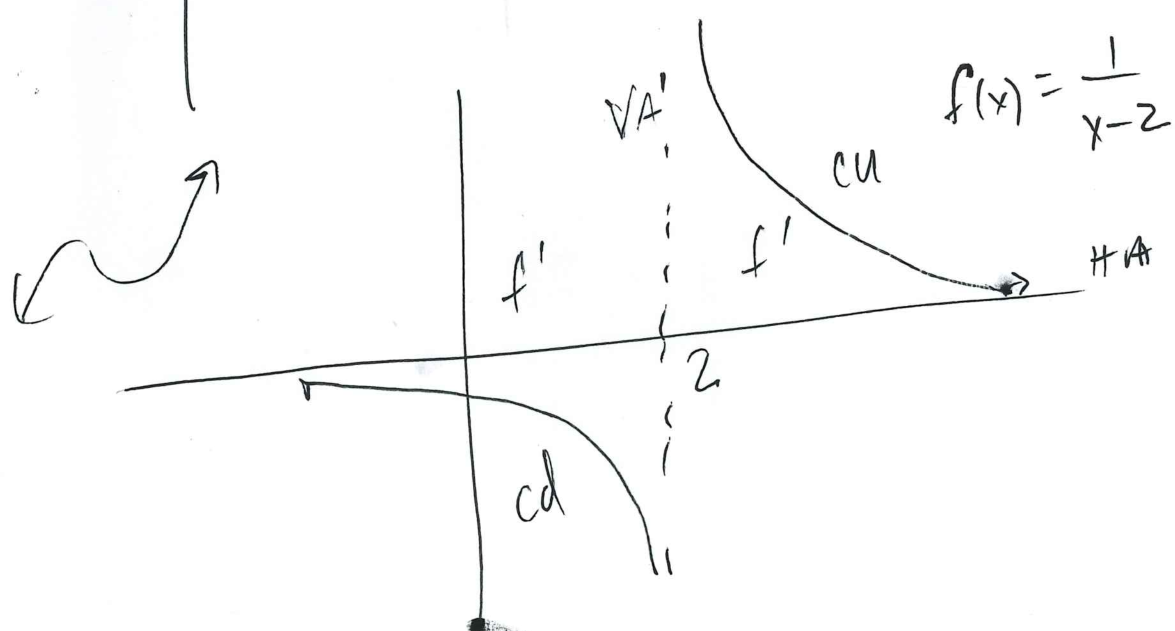
$-1 \quad 0 \quad 1$

$f''<0$ c.d. $\quad$ $f''<0$ c.d

Check to see what concavity does on either side of $x=0$

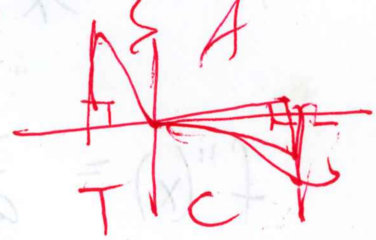
Since concavity does not change at $c=0$, then not POI





$\cos y + \sin x = -1 \quad \left(-\frac{\pi}{6}, \frac{2\pi}{3}\right)$

$-\frac{1}{2} + -\frac{1}{2} = -1$ ✓



$-\sin y \frac{dy}{dx} + \cos x = 0$

$$m = \frac{dy}{dx} = \left.\frac{\cos x}{\sin y}\right|_{\left(-\frac{\pi}{6}, \frac{2\pi}{3}\right)} = \frac{\cos \frac{-\pi}{6}}{\sin \frac{2\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}$$

$m = 1$

$y =$