LaTeX submissions are mandatory. The template for this problem can be found on the Piazza resource page for this course.

Problem 1
Use a determinant to identify all values of t and k such that the following matrix is singular. Assume that h and k must be real numbers.

$$A = \begin{bmatrix} 0 & 1 & t \\ -3 & 10 & 0 \\ 0 & 5 & k \end{bmatrix}$$

Solution:

Problem 2
Let $A = [\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}]$ be a 4×4 matrix whose determinant is equal to 2. What is the determinant of $B = [\boldsymbol{d}, \boldsymbol{b}, 3\boldsymbol{c}, \boldsymbol{a} + \boldsymbol{b}]$? Explain.

Solution:

Problem 3
By applying row operations to produce an upper triangular U, compute the following determinants:

1. $$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

2. $$A = \begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$$

Solution:

Problem 4
True or false, with reason if true and counterexample if false:

1. If A and B are identical except that $b_{11} = 2a_{11}$, then $\det(B) = 2\det(A)$.
2. The determinant is the product of the pivots.
3. If A is invertible and B is singular, then $A + B$ is invertible.
4. If A is invertible and B is singular, then AB is singular.
5. The determinant of $AB - BA$ is zero.

Solution:

Problem 5
A square ($n \times n$) matrix is called skew-symmetric (or antisymmetric) if $A^t = -A$. Prove that if A is skew-symmetric and n is odd, then $\det A = 0$. Is this true for even n?

Solution:

Problem 6
Find the determinant of an $n \times n$ matrix $A = I + J$ where I is the identity matrix and J is a matrix with all entries equal to 1.

Solution: