LaTeX submissions are mandatory. The template for this problem can be found on the Piazza resource page for this course.

Problem 1

Let $V = \mathbb{R}^5$ and let U be the subspace of V spanned by the vectors

$$\begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 2 \\ 1 \\ -2 \end{bmatrix},$$

and W the subspace of V spanned by the vectors

$$\begin{bmatrix} 3 \\ 2 \\ -3 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ -1 \\ -2 \\ 1 \end{bmatrix}.$$

Determine the dimension of $U \cap W$.

Solution:

Problem 2
Let

$$B = \begin{bmatrix} 1, & 2 \\ 0, & 1 \\ 1, & 0 \end{bmatrix}$$

What is the orthogonal projector P onto range(B) and what is the image under P of the vector $[1, 2, 3]^t$?

Solution:

Problem 3
If P is an orthogonal projector, then the matrix $I - 2P$ is orthogonal. Prove this algebraically, and try to give a geometric interpretation for the transformation represented by matrix $I - 2P$.

Solution: