LaTeXsubmissions are mandatory. The template for this problem can be found on the Piazza resource page for this course.

Problem 1
Find all vectors that are perpendicular to (1, 4, 4, 1) and (2, 9, 8, 2).

Solution:

Problem 2
From the nonorthogonal $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{a}_3$, find orthonormal vectors $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, $\boldsymbol{q}_3$.

$$\boldsymbol{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \boldsymbol{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \boldsymbol{a}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix},$$

Write matrix A as QR decomposition $A = QR$.

Solution:

Problem 3
Consider the matrices

$$A = \begin{bmatrix} 1, & 0 \\ 0, & 1 \\ 1, & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1, & 2 \\ 0, & 1 \\ 1, & 0 \end{bmatrix}$$

Answer the following questions by hand calculation.

(a) What is the orthogonal projector P onto range(A) and what is the image under P of the vector $[1, 2, 3]^t$?

(b) Same question for B.

Solution:

Problem 4
Let A be a real symmetric matrix. An *eigenvector* of matrix A is a non-zero vector x such that $Ax = \lambda x$ for some number λ which is called the *eigenvalue* corresponding to the eigenvector x.

Prove that if x and y are eigenvectors corresponding to distinct real eigenvalues λ_1 and λ_2, then x and y are orthogonal.

Solution:

Problem 5
Let u and v are two vectors in $\mathbb{R}^n$. The matrix $A = I + uv^*$ is known as a *rank-one perturbation of the identity*. Show that if A is nonsingular (that is, if it has an inverse), then its inverse has the form $A^{-1} = I + \alpha uv^*$ for some scalar α and give an expression for α. For what u and v is A singular? If it is singular, what is nullspace(A)?

Solution:

Problem 6
If P is an orthogonal projector, then the matrix $I - 2P$ is orthogonal. Prove this algebraically, and try to give a geometric interpretation for the transformation represented by matrix $I - 2P$.

Solution: