LaTeXsubmissions are mandatory. The template for this problem can be found on the Piazza resource page for this course.

Problem 1
Suppose the vector space V is the space of polynomials with real coefficients that have the degree ≤ 3. Use the basis $\{1, x, x^2, x^3\}$. In this basis, what is the matrix of the shift operator T that sends a polynomial $P(x) \to P(x+1)$?

Solution:

Problem 2
Let B be a 4 x 4 matrix to which we apply the following operations:

1. double column 1,
2. halve row 3,
3. add row 3 to row 1,
4. interchange columns 1 and 4,
5. subtract row 2 from each of the other rows,
6. replace column 4 by column 3,
7. delete column 1 (so that the column dimension is reduced by 1).

(a) Write the result as a product of eight matrices.
(b) Write it again as a product ABC (same B) of three matrices.

Solution:

Problem 3
Write the matrix $\left(\left((AB)^t\right)^{-1}\right)^t$ in terms of A^{-1} and B^{-1}.

Solution:

Problem 4
By using the reduction to the rref form, find the bases for the column space and nullspace of A and the solution to $Ax = b$:

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

Solution:

Problem 5
Let $f_1, \ldots f_8$ be a set of functions defined on the interval $[1, 8]$ with the property that for any numbers $d_1, \ldots, d_8$, there exists a set of coefficients $c_l, \ldots, c_8$ such that

$$\sum_{j=1}^{8} c_j f_j(i) = d_i, \quad i = 1, \ldots, 8.$$

(a) Show by appealing to the theorems of lecture 1 in Trefethen, Bau or to theorems in Lecture Notes that $d_1, \ldots, d_8$ determine $c_1, \ldots, c_8$ uniquely.
(b) Let A be the 8×8 matrix representing the linear mapping from data $d_1, \ldots, d_8$ to coefficients $c_1, \ldots, c_8$. What is the i, j entry of A^{-1}?

Solution: